Astro 507

Lecture 13

Feb. 24, 2020

Announcements:

- **Preflight 3 due Friday**: the CMB!
- Prodigal Instructor returns, thanks for your patience

In the distant past:

Robertson-Walker and relativistic cosmology

- re-derived redshift $z - a$ relation, and cosmic time dilation
- PS2: explored RW metric, introduced "conformal time"

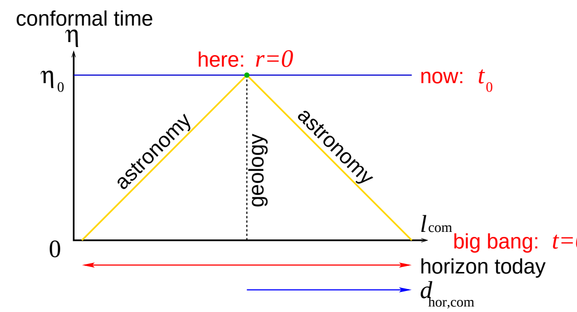
Today: last day of cosmological boot camp

Next time: apply tools to Dark Energy

Recap: Photon Propagation in FLRW

for a radial photon (i.e., coming to us)

$$d\ell_{\text{com}} = \frac{dr}{\sqrt{1 - \kappa r^2/R^2}} = \frac{dt}{a(t)} = d\eta$$



Why is η a "*conformal*" time?
conformal transformation = *angle-preserving*

$$ds^2 = a(\eta)^2 \; (d\eta^2 - d\ell_{\text{com}}^2) = a(\eta)^2 \; \times \text{(Minkowski form)}$$

preserves Minkowski "angles" in spacetime

$\to$ lightcones keep straight slopes: $d\eta/d\ell_{\text{com}} = 1$ on cone

compare photon trajectory in (t, ℓ_{com}) plane:

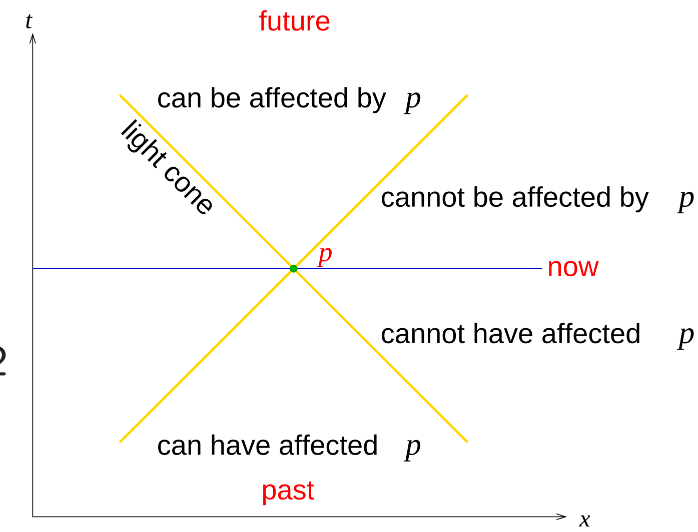
at early times: light cone "slope" $dt/d\ell_{\text{com}} = a(t) \ll 1$

Q: what does this look like? why inconvenient?

`www: light cones:` (t, ℓ_{com}) `vs` $(\eta, \ell_{\text{com}})$`plane`

Cosmic Causality

Recall special relativity (Minkowski space)
$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$



light: $ds = 0$ → *cone* $dt^2 = dx^2 + dy^2 + dz^2$

Now RW metric: $ds^2 = dt^2 - a^2 d\ell^2_{\rm com}$
introduce new time variable η: **conformal time**
defined by $d\eta = dt/a(t)$ (see PS2)

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\ell^2_{\rm com}\right)$$

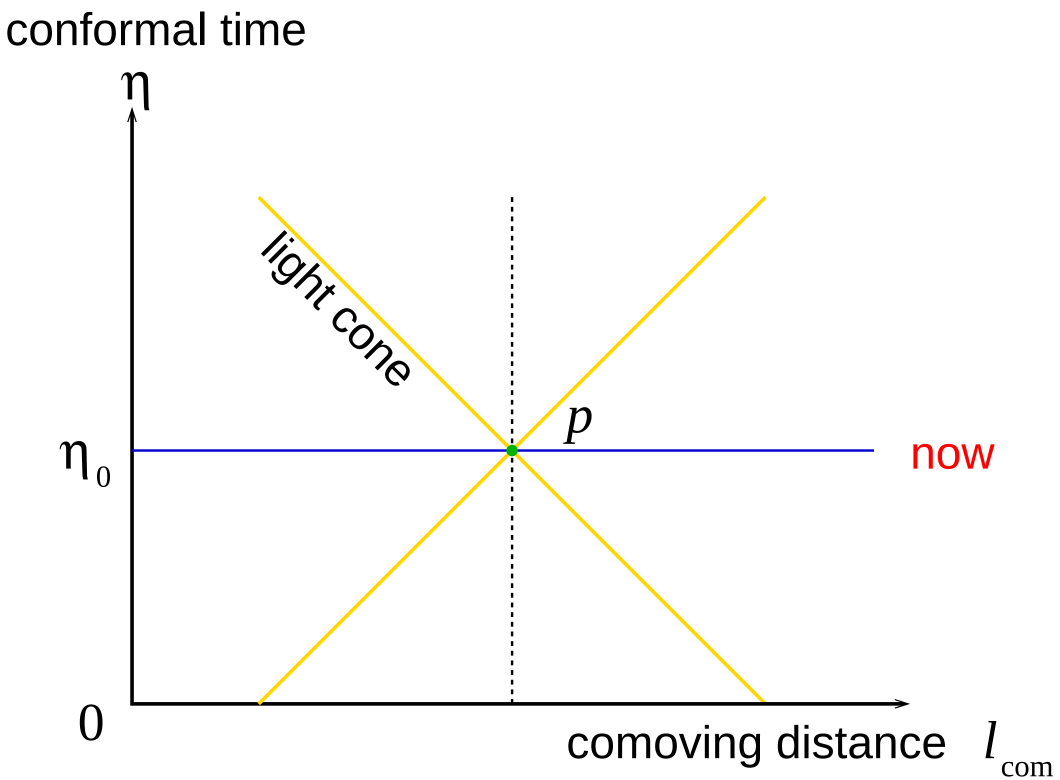
Q: implications?

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\ell^2_{\text{com}}\right) = a(\eta)^2 \ \times \ \text{(Minkowski structure)}$$

has same features as Minkowski space

$\Rightarrow$ *light cones still defined*

when use comoving lengths and conformal time



For a flat universe ($\kappa = 0$), it's even better:

$$ds^2 = a(\eta)^2 \left(d\eta^2 - dr_{\text{com}}^2\right) = a(\eta)^2 \ \times \ \text{(exact Minkowski form)}$$

In either case $\rightarrow$ spacelike, timelike, lightlike divisions same
and in $(\eta, \ell_{\text{com}})$ space:
light cone structure the same $\Rightarrow$ *causal structure the same*!

Namely:

- a spacetime point can only be influenced by events in past light cone
- a spacetime point can only influence events in future light cone

So far: like Minkowski
New cosmic twist: finite cosmic age
Q: implications for causality?

Causality: Particle Horizon

past light cone at t defined by
photon propagation over cosmic history:

$$\int_{t_{\rm em}=0}^{t_{\rm obs}=t_0} \frac{d\tau}{a(\tau)} = \int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} \equiv d_{\rm hor,com}(t_0)$$

where $d_{\rm hor,com}$ is the comoving distance
photon has traveled since big bang

if $d_{\rm hor,com} = \int_0^t d\tau/a(\tau)$ converges
then only a finite part of U has affected us
$\rightarrow$ $d_{\rm hor}$ defines *causal boundary*
$\rightarrow$ comoving "**particle horizon**"

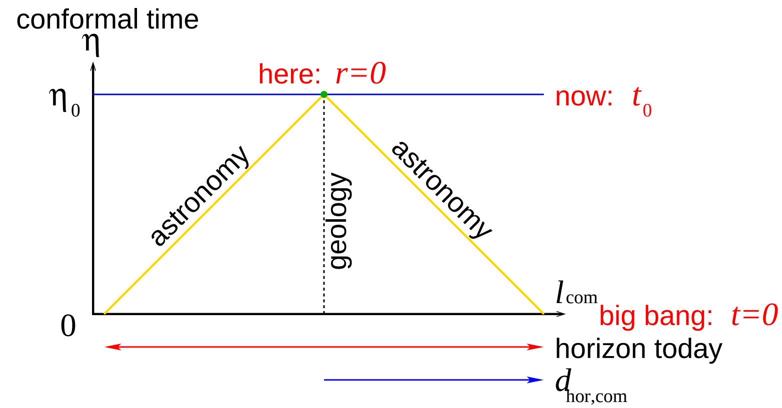
Q: physical implications of a particle horizon?
Q: role of finite age?
Q: sanity check–simple limiting case with obvious result?

Particle Horizons: Implications

our view of the Universe:

⋆ astronomical info comes from events along *past light cone*

⋆ *geological info* comes from *past world line*



if particle horizon finite (i.e., $\neq \infty$), then $d_{\text{horiz,com}}$:

- gives comoving size of **observable universe**
- encloses region which can communicate over cosmic time $\to$ causally connected region
- sets "zone of influence" over which particles can "notice" and/or affect each each other and local physical processes can "organize" themselves

e.g., shouldn't see bound structures large than particle horizon!

So *is* d_{hor} finite?

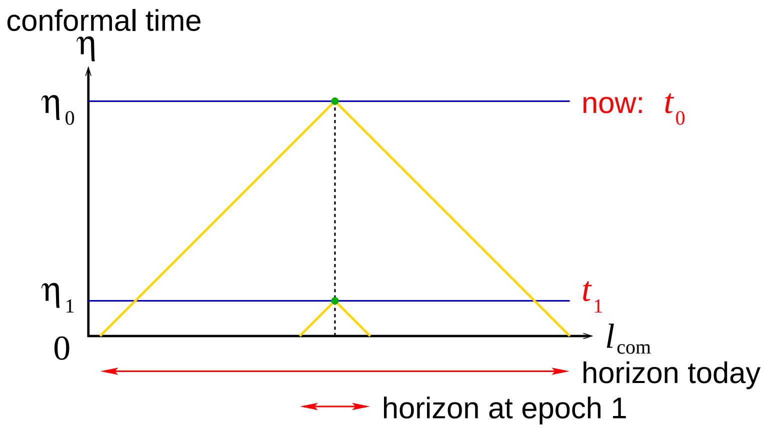
depends on details of $a(t)$ evolution as $t \to 0$:

behavior near singularity crucial

will see in PS3:

for matter, radiation domination:

- d_{hor} finite
- and $d_{\text{hor}} \to 0$ for $t \to 0$



Q: implications for CMB?

Hint: observed $T_{\text{CMB}}(\theta, \phi)$ isotropic to 5th decimal place...

will see in coming weeks

∞ ▷ inflation (if real!) adds twist!

Cosmic Distance Measures

More examples of how spacetime properties
impose relationships among observables

Warmup: Newtonian cosmology
another sanity check, limiting case
Q: validity range?

Consider Newtonian cosmo:

- given observed z, what is distance d_{Newt}?
- *Q: good for which z?*
- *Q: complications in full FLRW universe?*

"Newtonian Distance"

Newtonian cosmology:

- small speeds, weak gravity
  ignore curvature

Hubble's Law:

$$H_0 d_{\mathrm{Newt}} \equiv v \simeq cz \tag{1}$$

applicability: $z \ll 1$
solve:

$$d_{\mathrm{Newt}} = \frac{c}{H_0} z = d_H \; z$$

- naïve distance d_{Newt} is *linear in z*
- it is proportional to the Hubble length d_{H}
- fraction $d_{\mathrm{Newt}}/d_{\mathrm{H}} = z$; compare $t_{\mathrm{lookback}}/t_{\mathrm{H}} \approx z$

Distances and Relativity

Basic but crucial distinction, important to remember:

In *Newtonian/pre-Relativity* physics: space is *absolute*

- "distance" has unique, well-defined meaning:
  $\Rightarrow$ Euclidean separation between points
- can think of as "intrinsic" to objects and points

In *Special and General Relativity*: space *not* absolute

- distance observer-dependent, not intrinsic to objects, events
- different well-defined measurements can lead to different results for distance

In FLRW universe, "distance" not unique: answer depends on

- *what you measure*
- *how you measure it*

Proper Distance

So far: have constructed *comoving* coordinates
which expand with Universe ("home" of fundamental observers)

RW metric: encodes **proper distance**
i.e., *physical* separations as measured by metersticks/calipers:
▷ in RW frame i.e., by comoving observers=FOs
▷ *at one* fixed cosmic instant t

$$d\ell^2_{\rm prop} = a(t)^2 d\ell^2_{\rm com} = a(t)^2 \left(\frac{dr^2}{1-\kappa r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

Can read off proper distances for small displacements
as measured by FOs at time t:

- $d\ell_r^{\rm prop} = a(t)\, d\ell_r^{\rm com} = a(t)\, dr/\sqrt{1-\kappa r^2/R^2}$
- $d\ell_\theta^{\rm prop} = a(t)\, d\ell_\theta^{\rm com} = a(t)\, r d\theta$
- $d\ell_\phi^{\rm prop} = a(t)\, d\ell_\phi^{\rm com} = a(t)\, r \sin\theta d\phi$

Q: how to find distance for finite displacements?

for finite displacements: integrate small ones

e.g., radial distance (at t) between $r = 0$ and r is

$$\ell_r^{\text{prop}} = a(t)\ell_r^{\text{com}} = a(t) \int_0^r d\zeta / \sqrt{1 - \kappa\zeta^2/R^2} \qquad (2)$$

Note: $d\ell_r^{\text{prop}}/dt = \dot{a}\, \ell_r^{\text{com}} = H\, \ell_r^{\text{prop}}$ exactly!

$\rightarrow$ i.e., at a *fixed cosmic time* t

proper distance increase exactly obeys Hubble Law!

Q: what does this mean for points with $\ell_{\text{r}}^{\text{prop}} > d_H$*?*

Q: is this a problem?

Q: how would you in practice measure ℓ_r^{prop} *for large* r*?*

Apparent Brightness of a Standard Candle

consider a "**standard candle**"

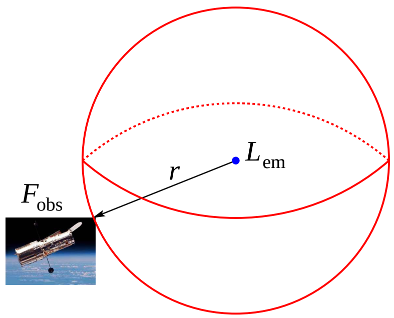
- object of known rest-frame luminosity

$$L_{\rm em} = \frac{dE_{\rm em}}{dt_{\rm em}}$$

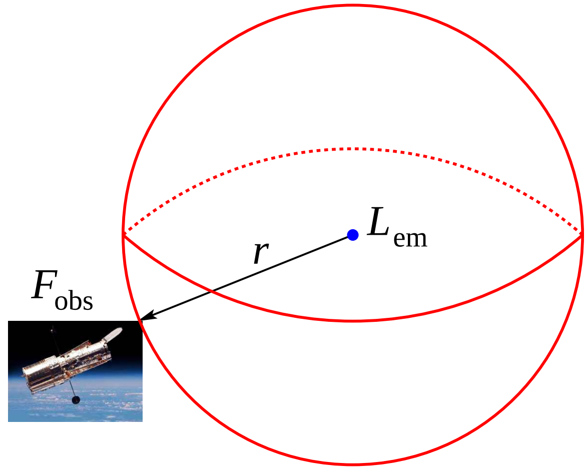
- emitting isotropically
- at epoch with $a_{\rm em}$ and at rest in cosmic frame
- also, assume no absorbing medium anywhere on sightline

if unresolved = **point source**, observables:

1. redshift $z_{\rm em}$
2. observed flux (apparent brightness)
   $F_{\rm obs} = dE_{\rm obs}/dt_{\rm obs}\ dA$
   summed over all wavelengths: "bolometric"



Q: Newtonian relation between L and F?



Goal: given std candle $L_{\rm em}$, want to relate observed $z_{\rm em}$ and $F_{\rm obs}$

$\Rightarrow$ find expression for **luminosity distance**

defined by Newtonian/Euclidean formula:

$$d_{\rm L}(z_{\rm em}) \equiv \sqrt{\frac{F_{\rm obs}}{4\pi L_{\rm em}}} \tag{3}$$

Q: effects in cosmological setting?

Strategy: start with observation, work back

Observation:

FO with telescope, area A_{det}
in time interval δt_{obs}
measures total energy $\delta\mathcal{E}_{\text{obs}}$; avg photon energy ϵ_{obs}

observed flux (bolometric, λ-integrated) given by

$$\delta\mathcal{E}_{\text{obs}} = F_{\text{obs}} A_{\text{det}} \delta t_{\text{obs}} \tag{4}$$

F_{obs} is rate of energy flow per unit area
as measured in observer frame

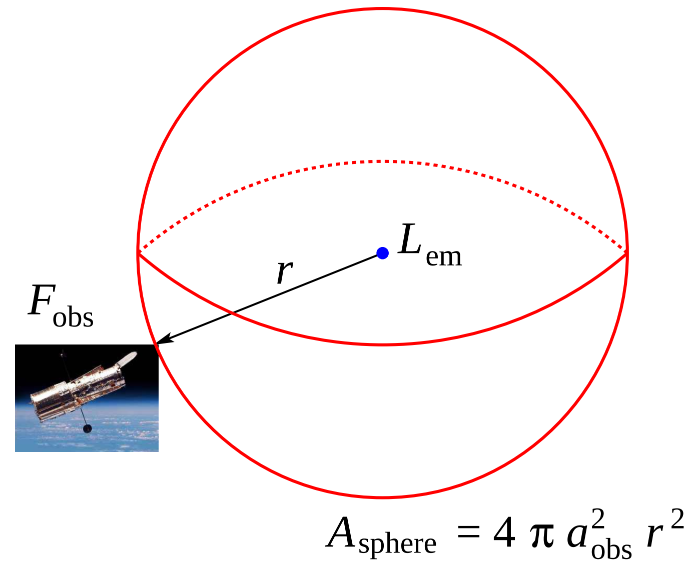
Q: what's invariant/observer independent as signal propagates?

Standard candle emitter:

luminosity L_{em} at $a_{\mathrm{em}}, z_{\mathrm{em}}$

with average photon energy ϵ_{em}

- choose $r_{\mathrm{em}} = 0$ as center
- light "cone" (sphere) today reaches us, has present area $A_{\mathrm{sph}} = 4\pi a_{\mathrm{obs}}^2 r^2 = 4\pi r^2$



key physical principle:

photon counts are invariant

i.e., all observers agree on how many detector registers

Q: how to quantify photon number conservation?