Astro 507

Lecture 14

Feb. 26, 2020

Announcements:

- **Preflight 3 due Friday**: the CMB!
- Office Hours after class (Instructor), tomorrow noon (TA)

Last time: cosmic distances

- lesson: in relativity distance are not unique or universal
  result depends on nature of measurement
- metersticks laid end-to-end give proper distance
  $\ell_{\rm prop}(t) = a(t)\ \ell_{\rm com}$
  changes at rate $\dot{\ell}_{\rm prop} = H\ell_{\rm prop}$: Hubble flow!
  reflects sum of small speeds between neighboring observers
  which move slowly, never close to c
  key scale: Hubble length $d_{\rm H} = H^{-1}$
  Q: physical significance?

Hubble Length

$d_{\mathsf{H}} = H^{-1}$, and comoving $d_{\mathsf{H,comov}} = d_{\mathsf{H}}/a = 1/aH$
measures physical and comoving distance where $\dot{\ell}_{\mathsf{prop}} = c$

physically:
boundary of cosmic region currently out of causal contact
estimate time until causally accessible: $t_{\mathsf{caus}} \sim d_{\mathsf{H}}/c = t_{\mathsf{H}}$

related to but diffent from comoving light travel distance at t:

$$d_{\mathsf{comov},\gamma}(t) \stackrel{\mathsf{flat}}{=} r(t) = \eta(t) = \int_0^t \frac{dt'}{a(t')} = \int_0^{a(t)} \frac{da}{a\ H} = \int_0^{a(t)} \frac{da}{a}\, d_{\mathsf{H}}(a)$$

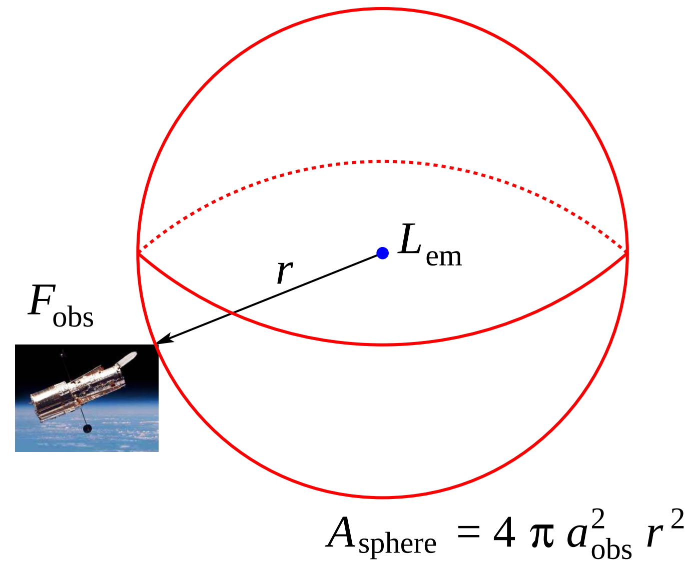
another key cosmic distance measure: **luminosity distance**
Q: ingredients? $d_L =$*? Newtonian results? Cosmo effects?*

Standard candle emitter:

luminosity $L_{\rm em}$ at $a_{\rm em}, z_{\rm em}$

with average photon energy $\epsilon_{\rm em}$

- choose $r_{\rm em} = 0$ as center
- light "cone" (sphere) today reaches us,
  has present area $A_{\rm sph} = 4\pi a_{\rm obs}^2 r^2 = 4\pi r^2$



key physical principle:

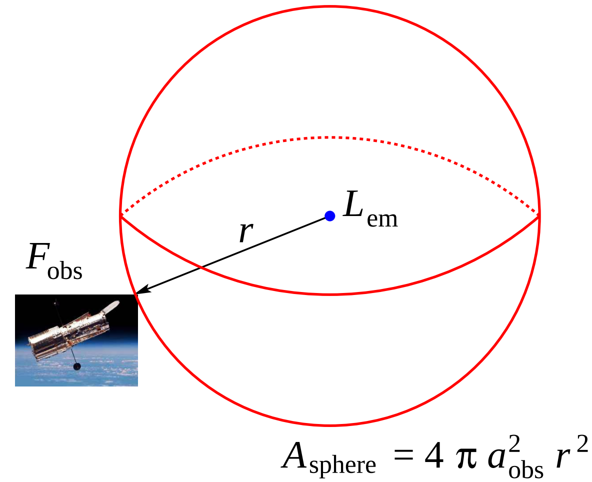
photon counts are invariant

i.e., all observers agree on how many detector registers

Q: how to quantify photon number conservation?

total photon counts through sphere at r:

$$\delta\mathcal{N}_{\mathrm{obs}} = \frac{F_{\mathrm{obs}} A_{\mathrm{sph}} \delta t_{\mathrm{obs}}}{\epsilon_{\mathrm{obs}}} = 4\pi r^2 \frac{F_{\mathrm{obs}}}{\epsilon_{\mathrm{obs}}} \delta t_{\mathrm{obs}}$$



total photon counts from source

$$\delta\mathcal{N}_{\mathrm{em}} = \frac{L_{\mathrm{em}}}{\epsilon_{\mathrm{obs}}} \delta t_{\mathrm{em}}$$

photon conservation: $\delta\mathcal{N}_{\mathrm{obs}} = \delta\mathcal{N}_{\mathrm{em}}$

$$F_{\mathrm{obs}} = \frac{\epsilon_{\mathrm{obs}}}{\epsilon_{\mathrm{em}}} \frac{\delta t_{\mathrm{em}}}{\delta t_{\mathrm{obs}}} \frac{L_{\mathrm{em}}}{4\pi r^2} \quad (1)$$

Q: and so?

$$F_{\rm obs} = \frac{\epsilon_{\rm obs}}{\epsilon_{\rm em}} \frac{\delta t_{\rm em}}{\delta t_{\rm obs}} \frac{L_{\rm em}}{4\pi r^2} \tag{2}$$

- energy redshifting $\epsilon_{\rm obs} = a_{\rm em}\epsilon_{\rm em}$
- time dilation $\delta t_{\rm obs} = \delta t_{\rm em}/a_{\rm em}$

So we have

$$F_{\rm obs} = a_{\rm em}^2 \frac{L_{\rm em}}{4\pi r^2} = \frac{L_{\rm em}}{4\pi(1+z)^2 r^2} \tag{3}$$

Q: and so?

Observed flux is

$$F_{\rm obs} = a_{\rm em}^2 \frac{L_{\rm em}}{4\pi r^2} = \frac{L_{\rm em}}{4\pi(1+z)^2 r^2} \tag{4}$$

identify **luminosity distance** via Newtonian/Euclidean result:

$$d_L \equiv \sqrt{\frac{L_{\rm em}}{4\pi F_{\rm obs}}} \tag{5}$$

and so

$$d_L = \frac{r}{a_{\rm em}} = (1+z)\ r$$

Q: why of practical observational interest?

Q: r unmeasured–how relate to observables?

Q: sanity checks? non-expanding? small z?

Q: why is $d_L \neq \ell_{\rm com}$?

Q: why is $d_L > r$?

Q: what if measure spectrum $F_\nu = dF/d\nu$?

luminosity distance: $d_L = (1+z)\ r(z)$

Note: relate r to emission redshift z via trusty photon propagation eq:

$$\int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} = \int_{t_{\rm em}}^{t_{\rm obs}} \frac{dt}{a(t)}$$

$$= \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a\dot{a}} = \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a^2 H(a)}$$

$$= \int_0^{z_{\rm em}} \frac{dz}{H(z)} = \eta(z_{\rm em})$$

where Friedmann gives $H(z)$

→ r and thus d_L manifestly depends on cosmology

(i.e., cosmic geometry, parameters)

★ d_L for SN Ia → cosmic acceleration!

Note: for alt radial variable χ

$d_L = (1+z)\ S_\kappa(\chi)$

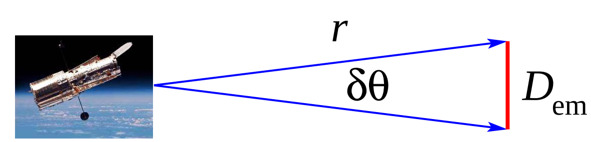
Extended Objects: Angular Diameter Distance

if object resolved as extended source on sky, not point source
then new observable available:

⋆ *angular size* $\delta\theta$

- and as usual, redshift z
  and flux (apparent bolometric brightness) F

input/assumption: "standard ruler"
known rest-frame size: diameter $D_{\rm em}$

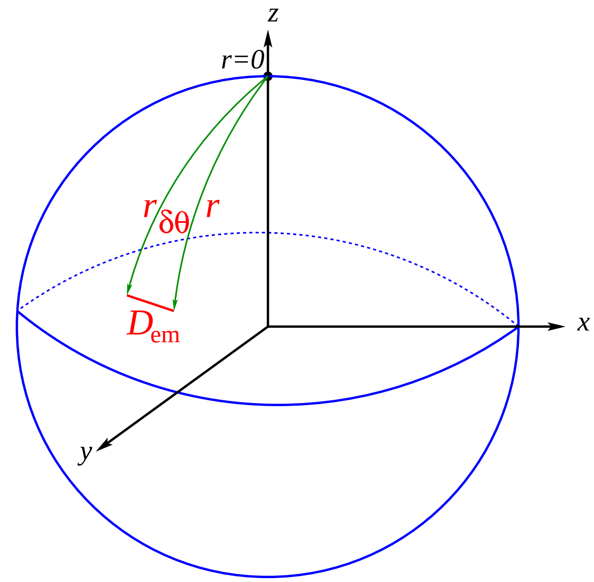


Goal: for std rulers, want to relate
observed z and $\delta\theta$

∞ *Q: effects in cosmological setting?*
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

To visualize, consider closed universe

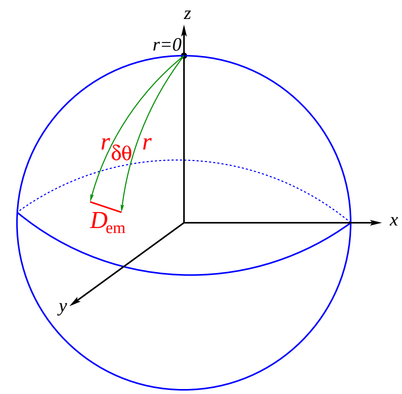
- observer at $r = 0$
- a pair of radial photons
  from edges of source
  trace longitudes



Invariant:
angular (longitude) separation $\delta\theta$ remains same
...while physical separation evolves, due to propagation
and cosmic expansion

At *emission* epoch, physical separation of photons
is standard ruler size $D_{\rm em}$
but also related to $\delta\theta$ and $r = r_{\rm em}$ via RW metric
Q: how?

At *emission* epoch, standard ruler size $D_{\rm em}$ at emission point r fixes angular separation $\delta\theta$:



$$D_{\rm em} = \delta\ell_\theta^{\rm prop,em} = a_{\rm em}\delta\ell_\theta^{\rm com} = a_{\rm em} r \delta\theta$$

But $\delta\theta$ remains fixed over propagation so today we observe

$$\delta\theta = \frac{D_{\rm em}}{a_{\rm em} r}$$

identify angular diameter distance via Newtonian/Euclidean result:

$$d_A \equiv \frac{D_{\rm em}}{\delta\theta} \tag{6}$$

and so

$$d_A = a_{\rm em} r = \frac{r}{1+z} = \frac{S_\kappa(\chi)}{1+z}$$

Angular diameter distance: $d_A = r(z)/(1+z)$

Q: why of practical observational interest?

Q: sanity checks?

Q: why is $d_A < r$?

Q: what if resolve at different λ?

Note:

- d_A depends on cosmological history via $r(z)$
- $d_A = a_{\rm em}^2 d_L = d_L/(1+z)^2$
  different measures!
  but ratio is cosmology independent

Q: implications for CMB fluctuations?

www: WMAP

Cosmic Acceleration & Dark Energy

Cosmic Conundrum: Observations vs Good Taste

1990's Cosmology:

- ▷ theory (Dicke coincidence *Q: whazzat?*, inflation), good taste, and some observational hints on large scales
  $\rightarrow \Omega_0 = 1$
- ▷ observation (e.g., galaxy halos, clusters) $\rightarrow \Omega_m \sim 0.3$

Q: possible reasons for discrepancy?

Q: observational tests?

Probing Cosmic Expansion as Far as the Eye Can See

Friedmann: cosmic *contents* control cosmic *dynamics*
$\rightarrow$ cosmic ingredients encoded in *history* of cosmic expansion

Strategy: **measure** $H(z)$ over large range in z

- Friedmann: $H = H(z; \Omega_0) \rightarrow$ data over large z range determine Ω_0
- alternatively, Friedmann accel:

$$H^2 = -2\frac{\ddot{a}}{a} - 8\pi GP - \frac{\kappa c^2}{R^2 a^2}$$

  $H(z)$ sensitive to acceleration, pressure, curvature

Q: what observables trace $H(z)$? what needed for large z range?

Supernovae as Standard Candles

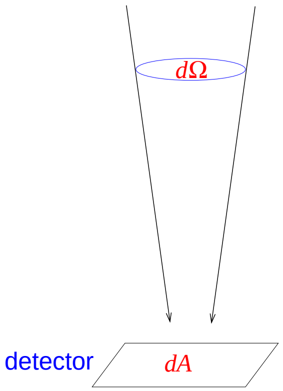
long "baseline" in $z \to$ requires **luminous** sources
supernova explosions–can outshine a galaxy
at peak, $L_{\rm SN,max} \sim 10^{10} L_\odot$
`www: SN 1994D; SN2014J in M82`

Procedure:
- identify SNe to use as standard candles
- measure flux F for events over wide range in z
- find $d_L(z) = \sqrt{L_{\rm SN}/4\pi F} \stackrel{\rm flat}{=} (1+z) \int_0^z dz/H(z)$
- infer $H(z) \to$ cosmic dynamics, parameters

First step:
all SN not created equal!
Q: what are basic SN classes observationally? how distinct physically?

Director's Cut Extras: Surface Brightness

Extended Objects Part Deux: Surface Brightness



if object is resolved, extended source on sky
can measure angular area and determine
surface brightness $I =$ flux/(angular area $\Delta\Omega$)

Q: physical effects: "normal" environment?
Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

Newtonian/Euclidean Surface Brightness

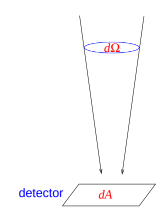
For intuition: review Newtonian/Euclidean result

- flat space
- no redshifting, time dilation

consider an **extended source**, i.e., not pointlike
which is **resolved** by your telescope
i.e., apparent angular size $>$ point spread function

observables:

- flux $F = dE/dt\ dA$ as before, but also
- angular dimensions $\rightarrow$ angular area $d\Omega$



Wavelength-integrated (bolometric) surface brightness
is wavelength-integrated flux per unit source angular area:

$$I_{\rm obs} = \frac{dE_{\rm obs}}{dA\ dt_{\rm obs}\ d\Omega} = \frac{dF_{\rm obs}}{d\Omega}$$

Dependence on source distance r?

- as usual, $F = L/4\pi r^2$
- source sky area $\Delta\Omega \Rightarrow$ physical area $S = r^2\Delta\Omega$, so

$$I_{\rm obs} = \frac{F_{\rm obs}}{\Delta\Omega} = \frac{L/4\pi r^2}{S/r^2} = \frac{L}{4\pi S}$$

Newtonian/Euclidean result *independent* of source distance!

"conservation of surface brightness"

fun consequences of surface brightness conservation:

- similar resolved, unobscured Galactic objects (e.g., nebulae) have similar surface brightness
- nearby large Galaxies have similar surface brightness to MW
- in daily life you rarely experience inverse square law e.g., brightness of faces of nearby vs distant classmates

Generalize to cosmological context: observed (bolometric) surface brightness

$$I_{\rm obs} = \frac{F_{\rm obs}}{\Delta\Omega_{\rm obs}}$$

1. already know $F_{\rm obs} = a_{\rm em}^2 L_{\rm em}/4\pi r^2$
2. RW metric says angular area

$$\Delta\Omega_{\rm obs} \simeq \frac{\delta\ell_\theta^2}{4\pi r^2} = \frac{D_{\rm em}^2}{4\pi a_{\rm em}^2 r} = \frac{A_{\rm em}}{4\pi a_{\rm em}^2 r^2}$$

Combine:

$$I_{\rm obs} = \frac{a_{\rm em}^2 L_{\rm em}/4\pi r^2}{4\pi A_{\rm em}/a_{\rm em}^2 r^2} = a_{\rm em}^4 \frac{L_{\rm em}}{A_{\rm em}} \quad (7)$$

$$= a_{\rm em}^4 I_{\rm em} = \frac{I_{\rm em}}{(1+z)^4} \quad (8)$$

Intensity of resolved, unobscured source at $z_{\rm em}$:

$$I_{\rm obs} = \frac{I_{\rm em}}{(1+z)^4}$$

- conservation of surface brightness" no longer true!
  vestige: no explicit dependence on r
- *cosmic dimming* $\propto (1+z)^4$
- dimming is independent of cosmology!
  useful consistency check!

Q: implications for CMB brightness?

CMB implications:

for blackbody, Stefan-Boltzmann sez

$$I = \frac{\sigma}{\pi} T^4$$

consider CMB, emitted at $z_{\rm em}$

with temperature $T_{\rm em}$

today, observe surface brightness

$$I_{\rm obs} = (1 + z_{\rm em})^{-4} I_{\rm em} = (1 + z_{\rm em})^{-4} \frac{\sigma}{\pi} T_{\rm em}^4 = \frac{\sigma}{\pi} \left(\frac{T_{\rm em}}{1 + z_{\rm em}} \right)^4$$

still follows blackbody law, but with

$$T_{\rm obs} = \frac{T_{\rm em}}{1 + z_{\rm em}}$$

which we have already derived by other means!