Astro 507
Lecture 16
March 1, 2020

Announcements:

- **Problem Set 3 posted, due next Friday March 6**
- Instructor F2F office hours 15 min after class Wed but online discussion available
- TA Office hours noon-1pm Thursday

Last time: evidence for acceleration
data: SN fainter (lower F) than in coasting/decelerating U

Today: possible interpretations

SN Ia Survey Observations

`www:  SNIa survey data`

★ luminosity distances show $d_L(\text{obs}) > d_L(\text{non}-\text{accel})$

★ standard candles **appear faint!**

in magnitudes, $m_{\text{obs}} > m_{\text{non}-\text{accel}}$

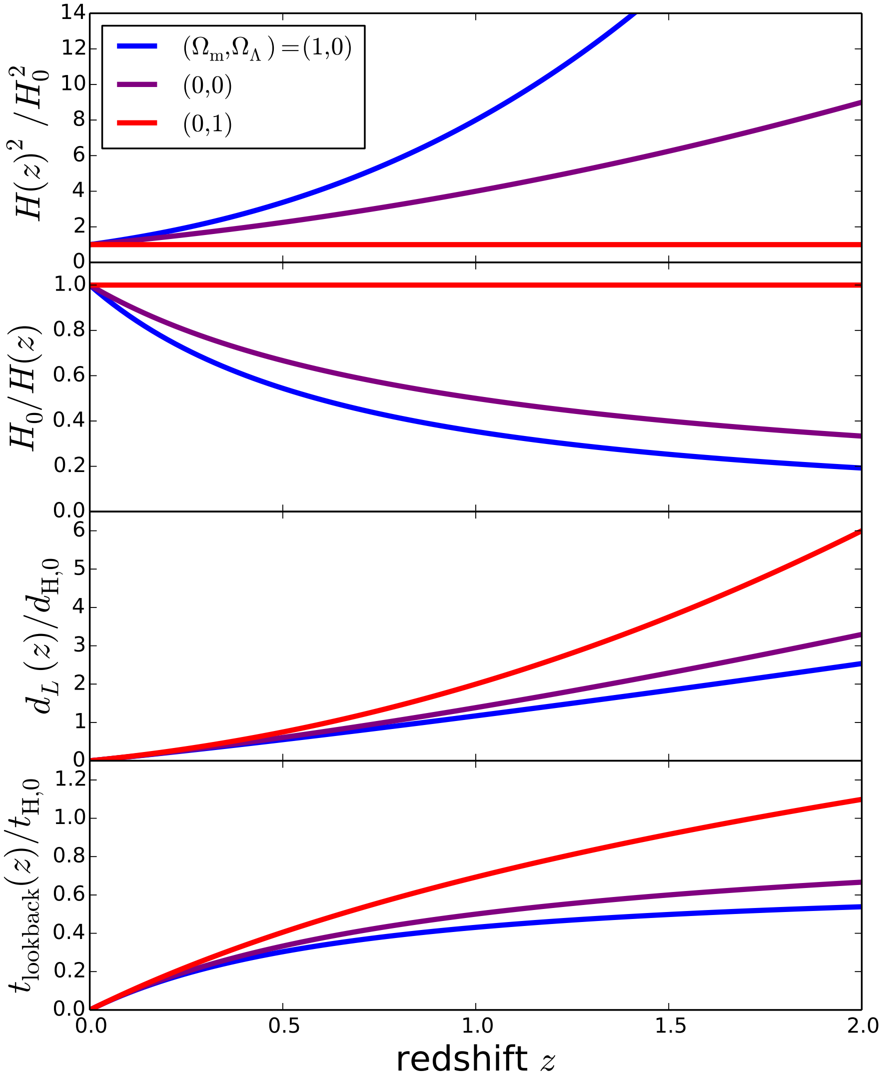
flux $F_{\text{obs}} < F_{\text{non}-\text{accel}}$

Why does acceleration give fainter candles than deceleration?

- standard candle measurements gives luminosity distance

$$d_L(z) = (1+z)\ \ell_{\text{comov}}(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

- for fixed z: fixed cosmic expansion during photon travel
- so higher $d_L \to$ higher photon comoving distance ℓ_{comov} during travel time, due to two effects *Q: guesses?*


$(\Omega_{\rm m},\Omega_\Lambda)=(1,0)$
$(0,0)$
$(0,1)$
$H(z)^2\,/H_0^2$
0
2
4
6
8
10
12
14
$H_0/H(z)$
0.0
0.2
0.4
0.6
0.8
1.0
$d_L(z)/d_{\rm H,0}$
0
1
2
3
4
5
6
$t_{\rm lookback}(z)/t_{\rm H,0}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0
redshift z

Faint Candles Point to Acceleration

$$d_L(z) = (1+z)\ \ell_{\mathrm{comov}}(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = (1+z)\int_{t_{\mathrm{lookback}}(z)}^{t_0} \frac{dt}{a(t)}$$

- photon travel time $t_{\mathrm{lookback}}(z)$ set by
  time Universe needs to expand by fixed amount
  least in declerating U, most in accelerating (fast/slow in past)
- also: photon *comoving* progress differs
  fast then slow in accelerating U: maximizes progress!

Q: possible explanations for faint supernovae/acceleration?
...(at least 3 distinct classes)
Q: pros and cons?
Q: how to observationally test?

Faint SN Ia: Whodunit?

⋆ Blame the Observations

maybe: SN Ia are *not* reliable standard(izable) candles
i.e., $m(\mathrm{obs}) \neq m(\mathrm{std\ candle})$
such that $L_{\mathrm{SN}}(\mathrm{high}z) < L_{\mathrm{SN}}(\mathrm{low}z)$ *systematically*

⋆ Blame Einstein

observations correct, but
expectations based on gravity theory = GR
maybe: GR incorrect/incomplete

⋆ Blame the Universe

observations correct, and GR correct as well, so
infer existence of new cosmic contents which create acceleration
  e.g., acceleration points to an accelerant!
maybe: Friedmann OK, but missing terms
  i.e., beyond matter (including DM!) and radiation
  new source(s) of ρ, P

What is to be done?

At face value
- SN Ia ⇒ U. is accelerating
- RW+Einstein ⇒ need new cosmic components

For now: assume these are true; then...

Our Mission

quantify–and ultimately identify–the new stuff
see if we can live with the consequences

But don't forget:
- ▷ keep checking SN Ia systematics
- ▷ don't dismiss gravity beyond Einstein:
  GR may itself be a limiting case of larger theory
  just as Newtonian gravity is limit of GR

First step:
Q: Friedmann–what are conditions for acceleration?

Acceleration in a FLRW Universe

Recall:

Cosmo principle (RW metric) + GR

= Friedmann

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{1}$$

But SNIa → $\ddot{a} > 0$:

$$P < -\frac{1}{3}\rho c^2$$

Q: implications? interpretation?

cosmic acceleration demands $\boxed{P < -\rho c^2/3}$

Cosmic pressure must be

⋆ non-negligible

⋆ **negative!** *Q: meaning?*

⋆ (for GR experts) violation of strong energy condition
$\rho + 3P \geq 0$ fails!

Exotic substance mandatory!

- NR matter and/or radiation in *any* form
  even weirdo particle dark matter (WIMPs, axions, ...)
  have $P \geq 0$: inadequate!
- new accelerant must be *dark*
  i.e., has not been undetected in EM radiation
- simplest solution is oldest...

Acceleration and the Cosmological Constant

Originally: Einstein modification of GR
to allow for *static* universe (PS3): $\ddot{a} = \dot{a} = 0$

- forced to introduce new constant of nature
  cosmological constant $\boxed{\Lambda}$
- $[\Lambda] = [\text{length}^{-2}]$; alters cosmic geometry
- spoils GR → Newtonian limit: instead,

$$\nabla^2 \phi = 4\pi G\rho - \frac{c^2}{3}\Lambda$$

Q: what does this do to Newtonian gravity?
Q: why isn't this immediately fatal?

Cosmo-Sociology: The Checkered History of Λ

Λ often invoked to solve cosmo problems,
then abandoned when observations improved

example: early measurements gave $H_0 \sim 500$ km s^{-1} Mpc^{-1}
$\rightarrow t_{\mathrm{H}} \sim 2$ Gyr $\ll$ age of Earth!
Lemaître (1931): Λ can give "loitering" Universe
quasi-static for a long time, then begins expanding recently

"My greatest blunder."

— A. Einstein, allegedly, on inventing Λ

"The cosmological constant is the last refuge of scoundrels."

— famous Chicago cosmologist and current Λ enthusiast, circa 1990

Living with Λ

With $\Lambda \neq 0$, new term in both Friedmann eqs

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2} + \frac{c^2}{3}\Lambda \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{3}$$

Note appearance & sign in acceleration

$\Rightarrow$ Λ an "accelerant" $\rightarrow$ "antigravity"

Q: intuitive reason? Hint: original purpose?

convenient to introduce $\Omega_\Lambda = \Lambda c^2/3H^2$

allows easy comparison of Λ term with others

Q: but you can guess which larger, based on observed accel?

The Data: Λ Emerges

SN Ia data in Λ cosmology:

- allow for $\Omega_\Lambda = \Lambda c^2/3H^2 \neq 0$
- find best fit to d_L data:

“concordance universe”

`www:` $\Omega_\Lambda - \Omega_\mathrm{m}$ `plane`

$$\boxed{\Omega_\Lambda \simeq 0.7 \qquad \Omega_\mathrm{m} \simeq 0.3} \tag{4}$$

This is amazing!

Q: why?

Λ Looms Large

acceleration demands $\Omega_\Lambda \sim 0.7$
roughly independent of CMB

- Einstein-de Sitter expectations of $\Omega_m = \Omega_0 = 1$
  totally ruled out!

- $\Omega_\Lambda \neq 0$: cosmo constant (or worse!) seems to exist!

- $\Omega_\Lambda \gtrsim 2\Omega_m$: U *dominated* by Λ *now*!

- *two mysteries seem related quantitatively:*
  CMB + galaxy clusters: $\Omega_0 - \Omega_m = \Omega_{other} \approx 0.7$
  SNe Ia: $\Omega_\Lambda \approx 0.7$
  a consistent picture of a bizarre universe!

Q: if this is all true, cosmic fate?

Λ and Cosmic Fate: Big Chill and Dark Sky

if acceleration is truly due to Λ then:

- already dominates Friedmann
- as a increases, matter & curvature terms drop
  $\rightarrow$ Λ dominates even more!

The bleak Λ-dominated future:

$\star$ future $a(t) \simeq e^{\sqrt{\Omega_\Lambda} H_0 (t-t_0)}$ $\rightarrow$ exponential expansion *forever*!
fate is not only *big chill* but *supercooling*

$\star$ *event horizon* exists: $d_{\text{event,comov}}(t_0) \simeq \Omega_\Lambda^{-1/2} d_H \sim 6400$ Mpc
we will *never* see beyond this!
worse still: later on,

$$d_{\text{event,comov}}(t_0 + \Delta t) = e^{-\sqrt{\Omega_\Lambda} H_0 \Delta t} d_{\text{event,comov}}(t_0)$$

event horizon shrinks exponentially with time!
$\rightarrow$ ever less to see!
observational astronomy from data mining only!

Λ as Vacuum Energy

Can rewrite Λ as energy density: ρ_Λ:
in Friedmann, put

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2} + \frac{\Lambda c^2}{3} \equiv \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{\kappa c^2}{R^2 a^2}$$

so that

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad \text{and} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\text{crit}}}$$

Then introduce pressure P_Λ in Fried accel:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda c^2}{3} \equiv -\frac{4\pi G}{3}(\rho + \rho_\Lambda + 3P + 3P_\Lambda)$$

can show:

$$P_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\rho_\Lambda$$

i.e., $P_\Lambda = w\rho_\Lambda$, with $w = -1$

Note:

- Λ is strict constant $\rightarrow$ ρ_Λ constant in space and time
  "energy density of the vacuum" $\rightarrow$ **dark energy**
- $P_\Lambda < 0$: as needed for acceleration
- equation of state parameter $w = -1$ preserves Λ constancy

So: Λ is equivalently a length scale
or an energy density
Q: what sets its value?

Dark Energy: Parameterized Ignorance

Theoretical Ignorance

No good (i.e., pre-existing) candidates for cosmic acceleration
unlike dark matter: high-E theory predicts stable exotic particles

Lacking guidance, look for general way to describe
cosmic substance responsible for acceleration: **dark energy**
recall: matter, radiation, Λ described by $P = w\rho c^2$
with w a constant

Write dark energy density and pressure with

$$P_{\mathrm{DE}} = w \ \rho_{\mathrm{DE}} c^2$$

"parameterize our ignorance" in $\boxed{w}$ (possibly not constant)
cosmo constant is limiting case *Q: Namely?*
Q: what can we say about w values?

Dark Energy: the Little We Know

What is w today?

In DE-only case

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = -\frac{4\pi G}{3}\rho(1 + 3w) \tag{5}$$

→ acceleration requires $w < -1/3$ today

Recall: cosmic first law is

$$d(\rho a^3) = -p\ d(a^3) = -w\rho\ d(a^3) \tag{6}$$

For constant w:

$$\rho_{\mathrm{DE}} \propto a^{-3(1+w)} \tag{7}$$

Q: sanity check–results for w = matter, radiation, Λ?
Q: connection between "w" dark energy and Λ?

Data: generalize Ω_Λ limits
to Ω_w and w (now two parameters)
`www: current limits`

$$\Omega_w \sim 0.7 \quad , \quad w < -0.76 \ (95\%\mathrm{CL})$$

- w close to -1: cosmo constant value!
- tests for w change weak but null
  → also like cosmo const!

What if w not constant?
Empirical approach: Taylor expand

$$w(a) = w_0 + w_a\,(1-a) \tag{8}$$

observations constraint parameters (w_0, w_a)
Q: does this allow for Λ result? if so how?
`www: present data`

Director’s Cut Extras

Λ and its Discontents

In Classical GR:

- ▷ Λ is a (optional) parameter to be measured
- ▷ no *a priori* insight as to its value
  (beyond escaping solar system limits)

But quantum mechanics & particle physics
offer a new perspective on vacuum energy

Recall: blackbody radiation
usually write total energy density:

$$\varepsilon_{\rm bb}(T) = \int \overline{n}\hbar\omega\,\frac{d^3p}{h^3} = \frac{1}{2\pi^2c^2}\int_{\omega=0}^{\infty}\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}\omega^2\,d\omega = a_{\rm Boltz}T^4$$

note that $\varepsilon \to 0$ as $T \to 0$: vacuum has no energy
...but (Λ aside) this was always a cheat!
Q: why? what omitted?

Uncertainty principle → nothing “at rest”
→ ground state “zero point motion”
→ zero point modes have energy $E_0 \neq 0$

Blackbody result: treats photon modes
as harmonic oscillators
but threw away zero point energy $E_0 = \hbar\omega/2$!
Cheated!

- handwaving excuse:
  E_0 cost of “assembling” oscillators/quanta
  ...and then only energy *differences* count
- in practice, usual Planck result is really
  $\varepsilon_{\mathrm{usual}} = \varepsilon_{\mathrm{tot}}(T) - \varepsilon_{T=0} = \varepsilon_{\mathrm{tot}}(T) - \varepsilon_{\mathrm{zeropoint}}$
- but in GR: curvature ↔ mass-energy density
  absolute energy scales matter!
  e.g., $(\dot{a}/a)^2 \sim 8\pi G/3 \ \varepsilon/c^2$

Q: what if we keep the zero-point energy?

Try keeping zero point energy:

$$\begin{aligned} \varepsilon &\sim \int_0^\infty \langle E(\omega)\rangle \; \omega^2 \; d\omega \qquad (9) \\ &= \int_0^\infty \left(\overline{n} + \frac{1}{2}\right) \hbar\omega \; \omega^2 \; d\omega \qquad (10) \\ &= \int_0^\infty \left(\frac{1}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}\right) \omega^3 \; d\omega \qquad (11) \\ &= \varepsilon_{\mathrm{usual}} + \varepsilon_{\mathrm{zeropoint}} \qquad (12) \end{aligned}$$

where the zero pont contribution is

$$\varepsilon_{\mathrm{zeropoint}} \sim \int_0^\infty \omega^3 \; d\omega = \infty^4$$

“ultraviolet catastrophe”!

Q: possible cures?

Vacuum Energy in Particle Physics

what is cause of catastrophe?

$$\varepsilon_{\text{zeropoint}} \sim \int_0^{\omega_{\text{max}}} \omega^3 \, d\omega \sim \omega_{\text{max}}^4$$

allowed $\omega_{\text{max}} \to \infty$

→ included modes of arbitrarily high energy
  arbitrarily small wavelength

If quanta *energy has upper limit* E_{max}
i.e., a minimum wavelength $\lambda_{\text{min}} = \hbar c / E_{\text{max}}$
then $\varepsilon_{\text{zeropoint}} \neq \infty$

Q: what might such a limit be?
Q: i.e., at what scale might energies "max out"?

The Planck Scale and Λ

Highest known energy scale in physics: **Planck Scale**
when *quantum effects become important for gravity*

a particle of mass m, energy mc^2
has quantum scale $\lambda_{\text{quantum}} = \hbar/mc$ (*Compton wavelength*)
equal to GR scale $\lambda_{\text{GR}} = 2Gm/c^2$ (*Schwarzchild radius*)
if $m = M_{\text{Pl}}$: the **Planck mass**

$$M_{\text{Pl}}c^2 = \sqrt{\frac{c}{G\hbar}}c^2 \sim 10^{19} \text{ GeV} \tag{13}$$

$$\ell_{\text{Pl}} = \frac{\hbar}{M_{\text{Pl}}c} \sim 10^{-33} \text{ cm} \tag{14}$$

if quanta have $E_{\text{max}} = M_{\text{Pl}}$ and $\lambda_{\text{min}} = \ell_{\text{Pl}}$
then estimate vacuum energy density

$$\rho_{\text{vac,Pl}} \sim M_{\text{Pl}}^4 \sim 10^{110} \text{ erg/cm}^3 \sim 10^{89} \text{ g/cm}^3$$

Q: implications?

Compare to the vacuum density in Λ:

$$\rho_{\text{vac,Pl}} \sim 10^{89}\ \text{g/cm}^3 \sim 10^{120} \rho_{\text{Lambda}}$$

mismatch is ~ 120 orders of magnitude!!

So the real question is not: *"Why have Λ at all?"*
but rather: *"Why isn't Λ gi-normous?"*

quantum gravity?
maybe some underlying symmetry set $\Lambda = 0$
  to avoid "fine-tuning" Λ
if so, then dark energy is not vacuum energy
  but some other energy density with negative pressure

high-energy phase transitions/symmetry breaking?
maybe symmetry breaking processes set vacuum energy
e.g., GUT, SUSY, electroweak, QCD
if so, how does each contribute to total vacuum?
run the numbers: best case is QCD

$$\varepsilon_{\mathrm{qcd}} \sim \Lambda_{qcd}^{4} \sim (100\ \mathrm{MeV})^{4} \sim 10^{30}\varepsilon_{\mathrm{dark\,energy}} \tag{15}$$

many orders of magnitude improvement, but not quite a fix!

Bottom line:
known quantum fields do not provide viable candidate
for source of vacuum energy $\rho_{\mathrm{vac}} = \rho_{\Lambda}$