Astro 507

Lecture 20

March 11, 2020

Announcements:

- **Preflight 4 due Friday noon**
- COVID-19 update: **don't come to class if you are sick**
  stay tuned for updates

Last time: cosmic microwave background radiation (CMB)

Q: summarize observed properties discussed so far?

The Isotropic CMB: Present Data

Spectrum

best data: FIRAS instrument on
Cosmic Background Explorer (COBE)
Fixsen et al (1996):

- intensity consistent with Planck form `www: data`

$$I_\nu = B_\nu(T_0) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\nu/kT_0} - 1}$$

- present all-sky temperature

$$\boxed{T_0 = 2.7255 \pm 0.0006 \text{ K}} \quad \text{COBE FIRAS}$$

- from Wien's law: spectral peaks are

$$\lambda_{\rm max} = \frac{0.290 \text{ cm K}}{T_0} = 1.06 \text{ mm} \quad (1)$$

$$\nu_{\rm max} = 58.5 \text{ GHz K}^{-1}\ T_0 = 159 \text{ GHz} \quad (2)$$

Q: what part of EM spectrum is this? relevant observatories?

Thermal Distortions: Chemical Potential

we will see: spectrum could be thermal but not Planck form
if so, would introduce "chemical potential" μ:

$$I_\nu = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT - \mu/T} - 1} \tag{3}$$

then $\mu/T < 9 \times 10^{-5}$
also can put limits on distortion by
superposition of blackbody spectra with different T

Polarization

zero on average, but small nonzero rms
Q: why can't there be a uniform polarization?
...more on this later

Q: what about redshifting effect on T? I? I_ν?

CMB redshifting:

- Wien says $\lambda_{\mathsf{max}} T = const$, and since $\lambda \propto a$,
  $T = T_0/a = (1+z)T_0$

- total (integrated) intensity $I = \sigma_{\mathsf{SB}} T^4/\pi$
  and thus observers at z would see $I(z) = I_0/a^4 = (1+z)^4 I_0$
  and conversely, $I_0 = I(z)/(1+z)^4$
  $\Rightarrow$ cosmological dimming of surface brightness
  (true for any I)

Q: what sets ε_γ? Ω_γ? n_γ?

Derived CMB Properties

the CMB is a blackbody, and thus:
all properties completely determined by temperature!

energy density

$$\varepsilon_{\gamma,0} = \frac{\pi^2}{15} \frac{(kT_0)^4}{(\hbar c)^3} = a_{\rm SB} T_0^4 = 0.26057 \ \mathrm{eV/cm^3} \quad (4)$$

evolves as $\varepsilon_\gamma = \varepsilon_{\gamma,0}/a^4 = (1+z)^4 \ \varepsilon_{\gamma,0}$
Q: c.f. starlight? `www: cosmic radiation backgrounds`

equivalent mass density

$$\rho_{\gamma,0} = \frac{\varepsilon_{\gamma,0}}{c^2} = 4.6451 \times 10^{-34} \ \mathrm{g/cm^3} \quad (5)$$

and thus

$$\Omega_{\gamma,0} = 5.04 \times 10^{-5} \ \left(\frac{0.7}{h}\right)^2 \quad (6)$$

number density

$$n_{\gamma,0} = \frac{2\zeta(3)}{\pi^2}\left(\frac{kT_0}{\hbar c}\right)^3 = 410.73 \text{ photons/cm}^3 \qquad (7)$$

with $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 = 1.202\ldots$

Q: is this a lot or a little? what's a useful comparison?

Q: physical implications of Planck form of CMB?

Planck Form: Implications

The observed CMB is consistent, at high precision, with
a purely Planckian form
that is: to high precision, **the CMB is a perfect blackbody**

but a blackbody spectrum:
- characterizes a system in thermodynamic equilibrium at T
- is independent of the size, shape, or composition of the system in equilibrium

thus the CMB implies that
the Universe once attained thermodynamic equilibrium
i.e., the Universe was once "*in good thermal contact*"
...we'll soon make this notion more precise

Note also that the *present* universe
must be *transparent* to the CMB

Q: why is this? what's the evidence?

Q: what does this imply about epoch probed by CMB?

The Present Universe is Transparent to the CMB

• high-z radio sources (quasars) observed: requires transparency
• CMB absorption seen in high-z atomic lines
• CMB scattering seen in galaxy clusters (SZ effect)
thus the CMB is now *decoupled* from cosmic matter
and originates beyond most distant observed sources $z \gtrsim 10$

thus: for at least $z \lesssim 10$, matter and radiation
in the Universe were *not held in equilibrium*

the equilibrium and thermalization needed to come earlier
• higher density
• higher temperature
the Planckian CMB points to a hot, dense early Universe

Q: what technology needed to calculate transparency?

For Radiation Transfer Fans

ignoring for now cosmological dimming
radiation transfer gives intensity change along sightline s:

$$\frac{dI_\nu}{ds} = -n_{\mathrm{abs}}\sigma_\nu \, I_\nu + j_\nu \tag{8}$$

due to absorbers with *number density* n_{abs}
and *absorption cross section* σ_ν
and sources with *emission coefficient* j_ν

as usual, rewrite as

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \tag{9}$$

with *optical depth* $d\tau_\nu = n_{\mathrm{abs}}\sigma_\nu \, ds$
and *source function* $S_\nu = j_\nu / n_{\mathrm{abs}}\sigma_\nu$

the solution to transfer equation has character

$$I_\nu \stackrel{\tau \gg 1}{\longrightarrow} S_\nu \stackrel{\text{thermal}}{\longrightarrow} B_\nu(T) \tag{10}$$

- *if sightline is optically thick* then
  observed intensity is source function, and furthermore
- *if source is thermal at T* then
  source function is Planckian

in other words:
a blackbody spectrum implies
an optically thick source in theromdynamic equilibrium

and so the Planckian CMB spectrum tell us
- *the Universe was once optically thick*
- *the Universe was once in thermodynamic equilibrium*

The Physics of the Isotropic CMB

We want to understand:

- what physics leads to the CMB?
- what cosmic epoch(s) does the CMB probe?
- what are the implications of the spectrum exquisitely good Planckian form?

Q: What is relevant physics?
Q: What are relevant cosmic ingredients?
Q: What are irrelevant (presumably?) cosmic ingredients?

The CMB as a Scattering Problem

recall: *any* observed photon has this life cycle:

- emission
- scattering (possibly none, possibly many times)
- absorption (i.e., detection)

thus: any *detected* = absorbed photon
points back to emission or most recent scattering event
e.g., daytime sky: Sun's emission disk vs off-source scattered blue light

the fact that the CMB is a *background*
to low-z objects $\rightarrow$ late-time U. is *transparent* to CMB

thus: the CMB probes exactly the epoch
when the universe was last able to scatter photons
i.e., the last time U. was *opaque* to its thermal photons

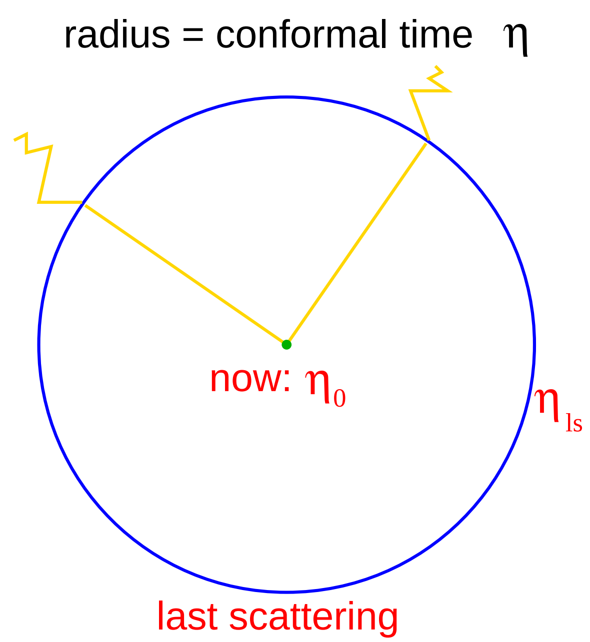
CMB as Cosmic "Baby Picture": Last Scattering Surface

CMB created by (and gives info about)
epoch of cosmic transition: *opaque* $\rightarrow$ *transparent*

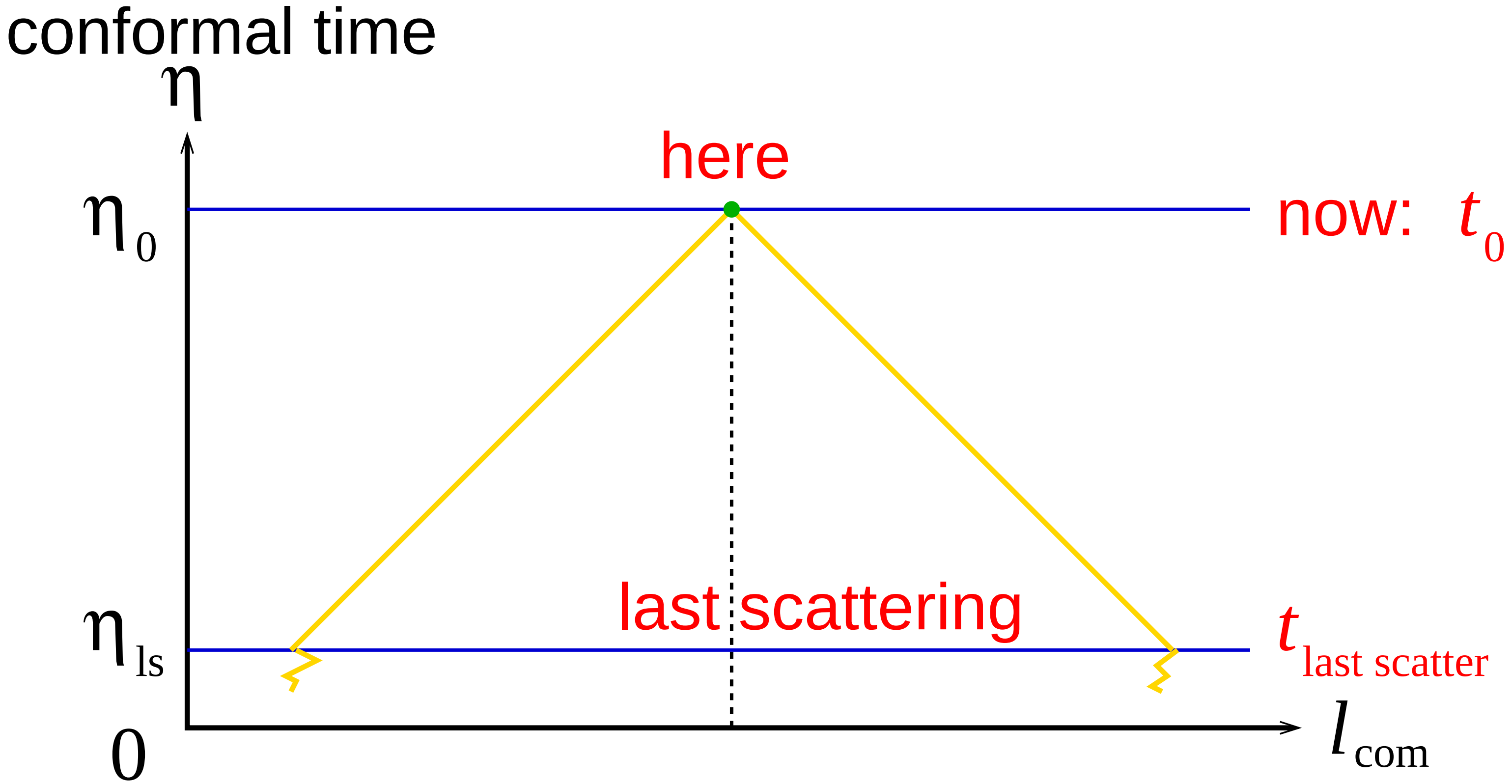
but transparent/opaque transition is
controlled by photon *scattering*

CMB "released" to "free stream"
at epoch of **"last scattering"** z_{ls}
$\rightarrow$ CMB sky map is a picture of U. then:
"**surface of last scattering**"

akin to photosphere of the Sun, but
"cosmic photosphere" is seen from inside!



Q: CMB photons in spacetime diagram?



For more detail, e.g., when is z_{ls}?

$\rightarrow$ need scattering technology

Highlights from Scattering 101

Collisions: $a + b \to$ stuff

Consider particle beam:

"projectiles," number density n_a

incident w/ velocity v

on targets of number density n_b

Due to interactions, targets and projectiles "see" each other
as spheres of projected area $\sigma_{ab}(v)$: the

cross section

⋆ fundamental measure interaction strength/probability

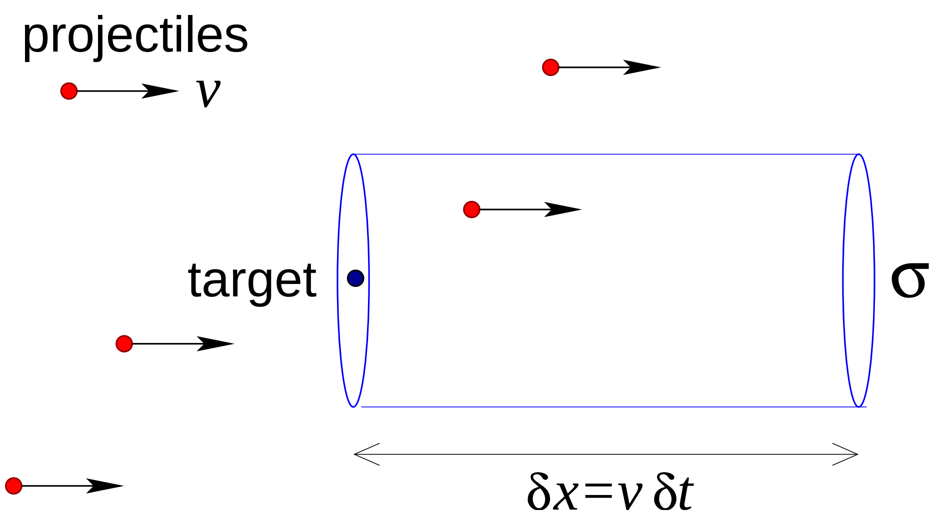
⋆ *atomic, nuke & particle physics meets astrophysics via* σ

in time δt, what is avg # collisions on one target?

Q: what defines "interaction zone" around target?

interaction zone: particles sweep out "scattering tube"

- *projectiles see targets as "bulls-eyes" of size* σ_{ab}
  ...and vice versa!
  sets tube cross-sectional area
- tube length $\delta x = v\delta t$



interaction volume swept around target:
$\delta V = \sigma_{ab}\delta x = \sigma_{ab} v \delta t$

collide: if a projectile is in the volume

Cross Section, Flux, and Collision Rate

in tube volume δV, # projectiles = $\mathcal{N}_{\text{proj}} = n_a \delta V$
so ave # collisions in δt:

$$\delta\mathcal{N}_{\text{coll}} = \mathcal{N}_{\text{proj}} = n_{\text{a}} \; \sigma_{ab} \; v \; \delta t \tag{11}$$

so $\delta\mathcal{N}_{\text{coll}}/\delta t$ gives

avg collision rate *per target* b $\boxed{\Gamma_{\text{per}\, b} = n_a \; \sigma_{ab} \; v = \sigma_{ab} \; j_a}$

where $j_a = n_a v$ is incident **flux**
Q: Γ units? sensible scalings n_a, σ_{ab}, v*? why no* n_b*?*

Q: average target collision time interval?
Q: average projectile distance traveled in this time?

estimate avg time between collisions on target b:

mean free time τ

collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$

so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\,b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\,b}} = \frac{1}{n_a \sigma_{ab} v} \tag{12}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_a \sigma_{ab}} \tag{13}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma_{ab}(E)$

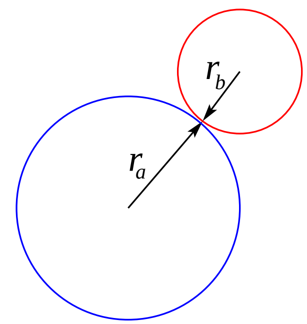
Q: physically, why the scalings with n, σ_{ab}?

Q: what sets σ for classical billiard balls?

Q: what set σ for $e^- + e^-$ scattering?

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., classical, macroscopic billiard balls)
then $\sigma \leftrightarrow$ particle radii:
$\sigma_{ab} = \pi(r_a + r_b)^2$



but: if interact by force field
(e.g., gravity, EM, nuke, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

Reaction Rate Per Volume

recall: collision rate *per target b* is $\Gamma_{\mathrm{per}\, b} = n_a \sigma_{ab} v$
total collision rate *per unit volume* is

$$r = \frac{dn_{\mathrm{coll}}}{dt} = \Gamma_{\mathrm{per}\, b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \, \sigma_{ab} \; v \qquad (14)$$

Kronecker δ_{ab}: 0 unless particles a & b identical

Note: volume rate *symmetric* w.r.t. the two particles
as it must be

What if particles have more than one relative velocity?