Astro 507

Lecture 21

March 13, 2020

Announcements:

- Welcome to Social Distancing Cosmology!
- Lecture in real time on Zoom, video posted after
- **Preflight 4 was due: thanks for great answers**
- **Problem Set 4 due Friday after break, March 27**

Last time: began the physics of the CMB

Q: Why cosmic? Why microwave? Why background?

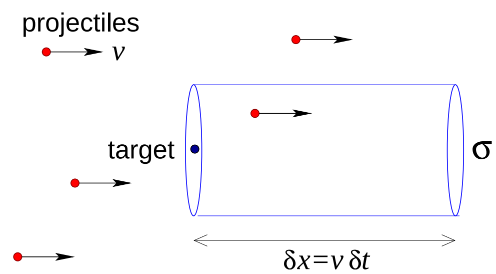
Q: where/when do CMB photons probe?

Q: physical significance of CMB sky image?

Q: when will an observer stop seeing the CMB?

Cross Section, Flux, and Collision Rate

projectiles a number density n_a
incident with speed v on targets b
with interaction cross section σ_{ab}



average collision rate *per target* b $\boxed{\Gamma_{\text{per}\,b} = n_a \, \sigma_{ab} \, v = \sigma_{ab} \, j_a}$

where $j_a = n_a v$ is incident **flux**

Q: average target collision time interval?
Q: average projectile distance traveled in this time?

estimate avg time between collisions on target b:

mean free time τ

collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$

so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\,b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\,b}} = \frac{1}{n_a \sigma_{ab} v} \tag{1}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_a \sigma_{ab}} \tag{2}$$

no explicit v dependence, but still $\ell(E) \propto 1/\sigma_{ab}(E)$

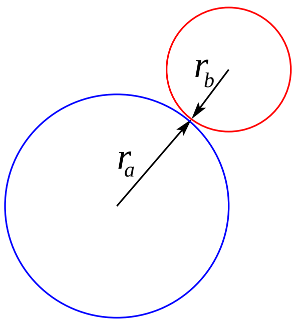
Q: physically, why the scalings with n, σ_{ab}?

Q: what sets σ for classical billiard balls?

 Q: what set σ for $e^- + e^-$ scattering?

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., classical, macroscopic billiard balls)
then $\sigma \leftrightarrow$ particle radii:
$\sigma_{ab} = \pi(r_a + r_b)^2$



but: if interact by force field
(e.g., gravity, EM, nuke, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

Reaction Rate Per Volume

recall: collision rate *per target b* is $\Gamma_{\mathrm{per}\,b} = n_a \sigma_{ab} v$
total collision rate *per unit volume* is

$$r = \frac{dn_{\mathrm{coll}}}{dt} = \Gamma_{\mathrm{per}\,b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \, \sigma_{ab} \, v \tag{3}$$

Kronecker δ_{ab}: 0 unless particles a & b identical

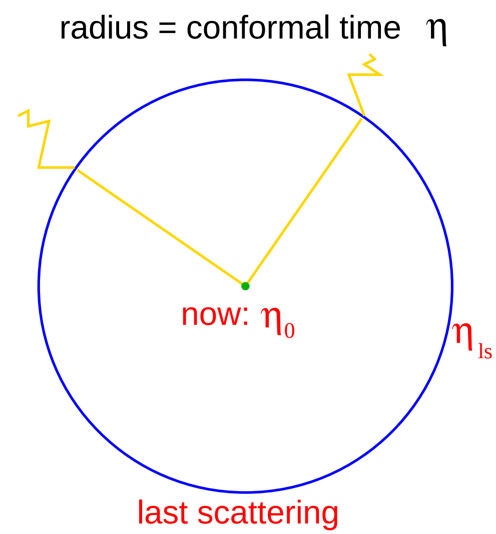
Note: volume rate *symmetric* w.r.t. the two particles
as it must be

What if particles have more than one relative velocity?

CMB: Last Scattering?

CMB is a background:
all other observed sources closer

- low-z Universe transparent to CMB photons
- CMB *scattering ineffective* for these z



But scattering rate $\Gamma(\mathrm{CMB}-\mathrm{matter})_{\mathrm{per}\gamma} = n_{\mathrm{targ}} c\sigma$

- low-z U. contains atomic matter = scatterers: $n_{\mathrm{targ}} > 0$
- photons can and do interact with atoms/ions/electrons: $\sigma > 0$

$\Rightarrow$ $\Gamma(\mathrm{CMB}-\mathrm{matter}) > 0$: *scattering must occur!*

Q: How can we reconcile these?
Q: Physical meaning, criterion for interaction "effectiveness"?

Particle Interactions in a FLRW Universe: Freezeouts

photon *decouple* plasma $\rightarrow$ CMB last scattering
when: expansion redshifting & volume dilution stops interactions

$$\Gamma_{\rm scatter} \lesssim H \tag{4}$$

or mean free time "infinite" $\rightarrow \tau \gtrsim t_H \sim t$
or mean free path "infinite" $\rightarrow \ell > d_{\rm hor,phys}$
Q: which of these is best to use?

⋆ This criterion of very *general cosmological importance* including CMB but also all of Early Universe!

⋆ Since Γ depends on particle energies $\rightarrow T$
and usually Γ *increases* (strongly) with T
$\Gamma \lesssim H$ sometimes known as condition for "**freezeout**"

⋆ *freezeouts a central aspect of much of cosmology*
CMB, big bang nuke, particle dark matter, 21 cm, ...

CMB Epoch: Freezeout of Cosmic Photon Scattering

Our Mission determine CMB release epoch
to do this: need photon scattering in cosmic environments

free electrons scatter photons
at low energies, cross section constant: Thomson

$$\sigma_{e\gamma} = \sigma_T = \text{const} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 0.665 \times 10^{-24}\ \text{cm}^2$$

Q: p has same charge–why can we ignore $p-\gamma$ scattering?

Q: what is scattering rate per photon?

CMB Epoch: Egregiously Naïve Treatment

- present baryon density $n_B \approx n_e$ total electron density *Q: why?* evolves as $n_e = n_{e,0}\ a^{-3}$
- using this, evaluate scattering rate *per photon*

$$\Gamma_\gamma = n_e \sigma_T c \stackrel{\text{naïve}}{=} n_{e,0} \sigma_T c\ a^{-3} \sim 5 \times 10^{-21}\ \text{s}^{-1}\ a^{-3} \tag{5}$$

- expansion rate evolves roughly as matter-dom: $H = H_0 a^{-3/2}$

compare scattering and expansion rates:

$$\frac{\Gamma_\gamma}{H} \stackrel{\text{naïve}}{\simeq} 2 \times 10^{-3} a^{-3/2} = 2 \times 10^{-3} (1+z)^{3/2} \tag{6}$$

Q: implications of $z = 0$ value?

- this would imply $\Gamma_\gamma > H$ when $z \gtrsim 60$
  Q: what is qualitatively promising about this?
- quantitatively, this is **wrong**: $z_{\text{last scatter}} \gg 60$
  Q: where did we go wrong?

U. mostly composed of diffuse (gaseous) matter

Q: what are possible states of this matter?

Q: how does each interact with photons?

Q: which absorbs/scatters the most, least efficiently?

`Demo: flame in projector beam`
Q: brighter or darker?
Q: why do we get the result we do?

Photon Scattering Agents

Photon scatter off of charged matter: atoms, ions, electrons
mostly H (90% by number, 75% by mass)
rest is mostly He, then traces of others

molecules: H_2 essentially invisible *Q: why?*

neutral atoms: "H I" – essentially invisible
unless $E_\gamma =$ level difference, e.g., $E(\text{Ly}\alpha) = E_2 - E_1 = 10.2$ eV
or $E_\gamma > 13.6$ eV binding

ionized gas/plasma: *free e^- readily scatter photons* $e\gamma \to e\gamma$
at low energy $E_\gamma \ll m_e c^2$, Thomson scattering
$\sigma_{e\gamma} = \sigma_T$

Q: lesson for CMB

The CMB and Recombination

In cosmic matter, photon scattering controlled
by availability of **free electrons** – bound e don't count!
▷ ionized U: e^- abundant, scattering rapid
▷ neutral U: H essentially transparent to thermal background

ionized $\leftrightarrow$ opaque
neutral $\leftrightarrow$ transparent

CMB originates in **(re)combination**
in transition $p + e \rightarrow \mathrm{H} + \gamma$ "the fog clears"
- plasma $\rightarrow$ neutral H
- photon last scattering $\rightarrow$ free streaming
- drunken stagger $\rightarrow$ sober sprint

Q: what (directly) determines when photons decouple *from plasma?*
Q: how is recombination different from decoupling? related?

Recombination and Decoupling

decoupling set by *freezeout* of scattering
as seen by photons → when $\Gamma_{\text{scatter,per}\gamma} \lesssim H$
U. transition: opaque → transparent
sets *"cosmic photosphere"* at which CMB released

(re)combination is when $p + e \to \text{H} + \gamma$
U. transition: ionized → neutral

these are *logically and physical distinct* epochs
but close in time and physically *related*:
photon scattering dominated by *free* e^-: *Thomson scattering*
and free e^- abundance drops enormously at recombination
→ recombination leads to decoupling

Q: pre-decoupling, what should photon spectrum be?
Q: how are photon, plasma temperatures related?

Cosmic Thomson Scattering

Pre-decoupled photons in thermal equilib with plasma
$\rightarrow$ initially I_ν is Planck spectrum, $T_\gamma = T_e$
Thomson scattering continues until free e gone

Fun facts about Thomson scattering $e\gamma \rightarrow e\gamma$
- ▷ interaction strength *energy-independent*: σ_{T} a constant
- ▷ an *elastic* process: photon energy essentially *unchanged*
- ▷ a "two-to-two" reaction: photon number *conserved*
- ▷ scattering *anisotropic* relative to initial photon direction
  angular distribution (scattering per solid angle $d\Omega$)

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{e^2}{m_e c^2}\right)^2 \left(1 + \cos^2\theta\right) \quad (7)$$

  includes a *quadrupole* component $\rightarrow$ creates polarization!

Assume that recomb is a freezeout *only* of Thomson:
Q: implications for post-recomb (i.e., observed) CMB spectrum?

CMB Spectrum: The Magic of Thomson Scattering

Thomson implications for cosmic last scattering:

- $\sigma_{\rm T}$ energy-indep $\rightarrow$ simultaneous freezeout at all freq ν
- elastic scattering $\rightarrow$ no change in spectral *shape*
  only changes photon directions
- photon number cons $\rightarrow$ don't add or subtract to spectrum
- anisotropic scattering w.r.t. initial photon direction
  but *if* initial directions isotropic $\rightarrow$ no net anisotropy created

magic of Thomson scattering:

⋆ simultaneous freezeout of all photons (all ν)

⋆ photon spectrum *preserved*

Q: implications of observed Planckian CMB spectrum?

Q: implication of number conservation of Thomson/Compton?

The CMB Demands a Hot Big Bang

observe *thermal* (Planck) CMB spectrum today
$\Rightarrow$ *thermal* CMB spectrum *pre*-decoupling!
$\Rightarrow$ in early U: photons thermalized, coupled to matter!

Cosmic matter & radiation once in "good thermal contact"
$\rightarrow$ but this requires much higher T, ρ than seen today
$\rightarrow$ CMB demands Universe went through *hot*, *dense* early phase
$\Rightarrow$ CMB $\rightarrow$ *hot big bang*

Compton/Thomson scattering conserves photon number
but Planck spectrum has fixed number density at T
$\Rightarrow$ **early Universe needed photon number-changing processes**
e.g., bremsstrahlung $e + \text{nucleus} \rightarrow e + \text{nucleus} + \gamma$
moreover: we will see that $n_\gamma \sim 10^9 \ n_{\text{baryon}}$
$\Rightarrow$ need huge photon source! *Q: ideas?*

Q: real-Universe complications?

in the real Universe, non-Thomson processes operate

most notably: as recombination begins, *neutral H* present
resonant emission and absorption due to H lines
does lead to *non-thermal distortions* in CMB

but turns out distortions are at high frequency
i.e., nonthermal perturbations expected to be significant
only at $h\nu \gtrsim 40kT$
why this scale? we will see...

Last Scattering: Including Recombination

Recombination Revisited

For simplicity, we will assume baryons are only protons

`www: laboratory hydrogen plasma`

and will consider only Thomson scattering (excellent approx!)

Then: scattering rate per photon is

$$\Gamma_\gamma = n_{e,\mathrm{free}} \sigma_\mathrm{T} c \;\propto\; n_{e,\mathrm{free}} \tag{8}$$

and last scattering when $\boxed{\Gamma_\gamma \simeq H}$

last scattering/decoupling controlled by *free electron density*

$n_{e,\mathrm{free}}$ changes due to

- cosmic volume expansion $\propto a^{-3}$
- recombination: free e^- lost to neutral H

rewrite to account for each $n_{e,\text{free}}$ effect separately:

$$n_{e,\text{free}} = X_e n_{e,\text{tot}} = X_e n_{\text{baryon}} \tag{9}$$

• baryon density $n_{\text{b}} \propto a^{-3} \propto T^3$
gives volume dilution

• "ionization fraction"

$$X_e \equiv \frac{n_{e,\text{free}}}{n_{e,\text{free}} + n_{e,\text{bound}}} = \frac{n_p}{n_p + n_{\text{H}}} = \frac{n_p}{n_{\text{b}}} \tag{10}$$

unchanged by volume dilution
only depends on recombination thermodynamics:
i.e., $X_e = X_e(T) = X_e(z)$

in homogeneous U
Q: what changes photon number density after recombination?
Q: what changes spectrum after recombination?
Q: naïve estimate of recombination $T_{\text{rec}}, z_{\text{rec}}$?
Q: zeroth-order treatment of $X_e(T)$?

Recombination: Improved Naïve View

Given hydrogen binding energy

$$B_{\mathrm{H}} = E(p) + E(e) - E(\mathrm{H}) = 13.6 \text{ eV}$$

simple estimate of recomb epoch goes like this:

Binding sets energy scale, so
★ when particle energies above B_{H}: U ionized,
★ otherwise: U neutral
→ naïvely expect transition at $T_{\mathrm{rec,naive}} = B_h \sim 150{,}000$ K

But we know $T = T_0/a$, so estimate recomb at

$$\left.\begin{array}{rcll} a_{\mathrm{rec,naive}} & = & \frac{T_0}{T_{\mathrm{rec,naive}}} & \sim 2 \times 10^{-5} \\ z_{\mathrm{rec,naive}} & = & \frac{T_{\mathrm{rec,naive}}}{T_0} - 1 & \sim 50{,}000 \end{array}\right\} \text{wrong!}$$

Q: guesses as to what's wrong?
Q: how to do this right?