Astro 507
Lecture 23
March 23, 2020

Announcements:

- **Problem Set 4 due Friday**
- Office Hours (online):
  Instructor: Wed 3-4pm, Fri 3-4pm
  TA: Thu noon-1pm

Cosmology in Quarantine: Staying Connected

we are inventing a new way to do graduate study
interaction more challenging but more important

- questions/answers: can unmute, or now use chat
  Ada will read chat responses
- it's good to see your faces!
  you are welcome to show your video if you want
- if you have ideas on how to improve class, tell me!

Last Time: Recombination and the Mighty Saha Equation

CMB decoupling: freezeout of Tompson scattering $e\gamma \to e\gamma$
freezeout condition $H = \Gamma_{\mathrm{T}} = n_{e,\mathrm{free}} \, \sigma_{\mathrm{T}} \, c \quad \propto n_{e,\mathrm{free}}$
free electrons: $n_{e,\mathrm{free}} = X_e \; n_{e,\mathrm{tot}} \approx X_e n_{\mathrm{B}} = X_e \eta n_\gamma$
with $\eta = n_{\mathrm{B}}/n_\gamma \approx 6 \times 10^{-10}$

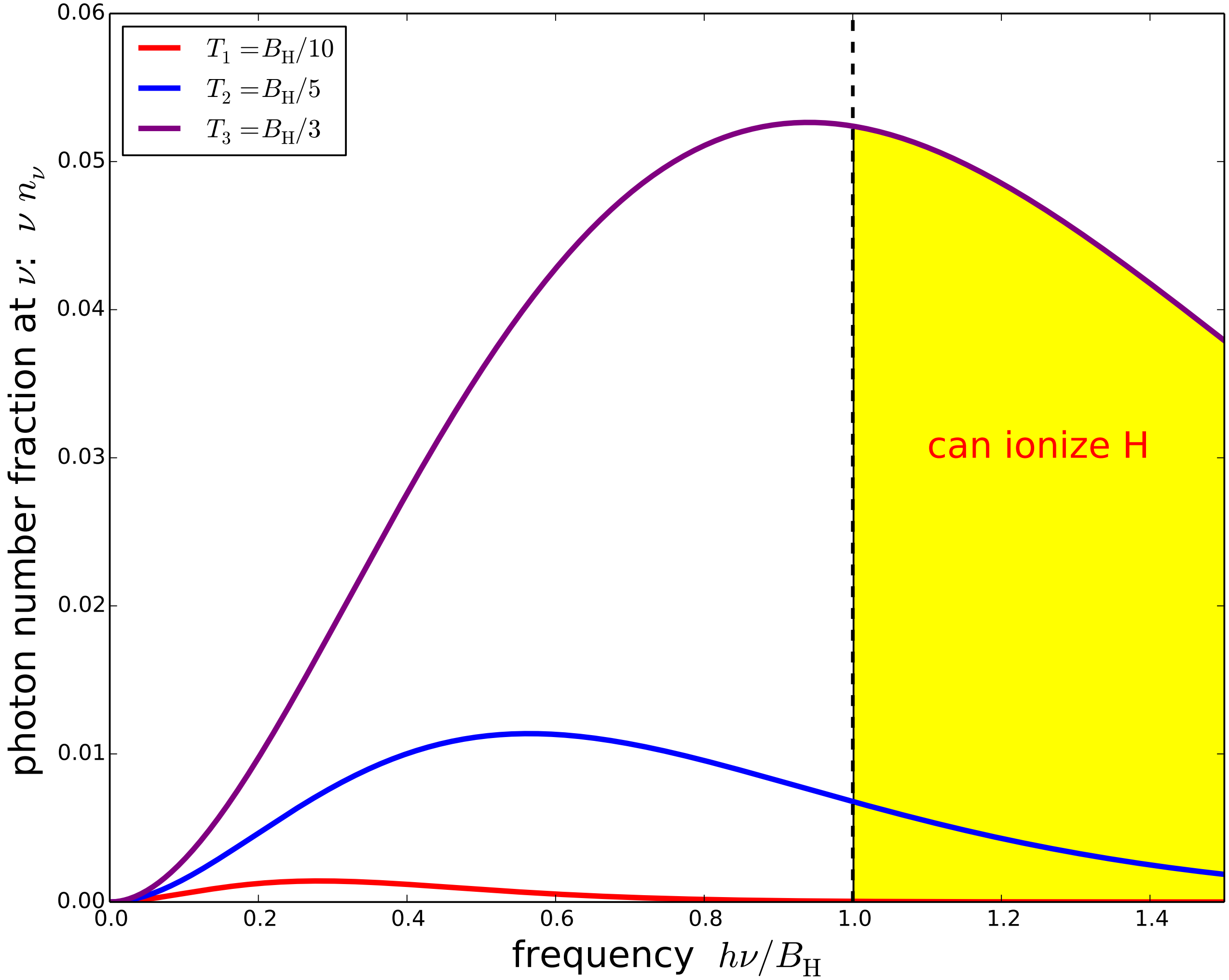
ionization fraction controlled by **Saha equation**

$$\frac{n_e n_p}{n_{\mathrm{H}}} \approx \frac{X_e^2}{1 - X_e} n_{\mathrm{B}} \;=\; \frac{g_e g_p}{g_{\mathrm{H}}} \left(\frac{m_e m_p}{m_{\mathrm{H}}} \right)^{3/2} \left(\frac{T}{2\pi\hbar^2} \right)^{3/2} e^{-(m_e + m_p - m_{\mathrm{H}})/T} \quad (1)$$

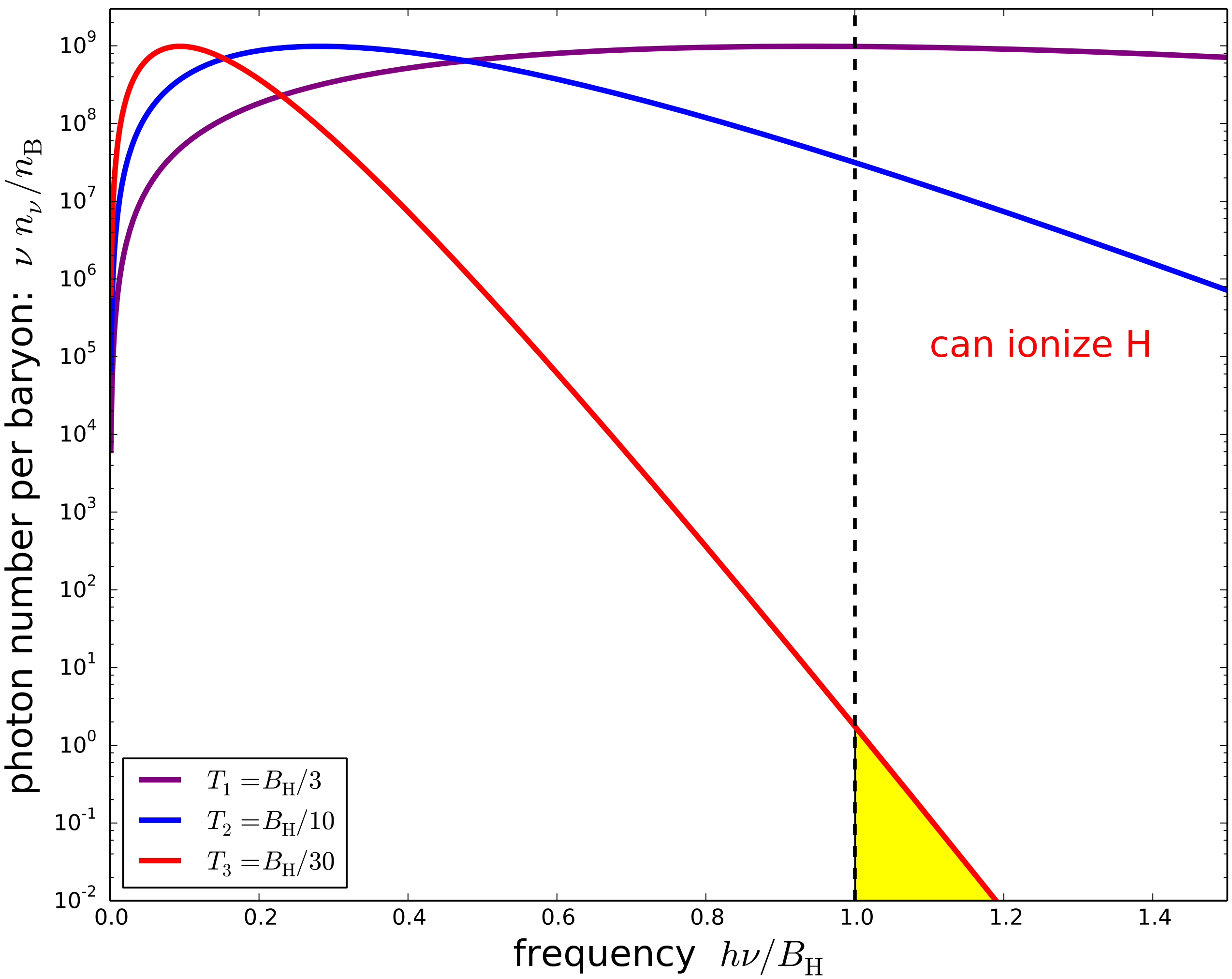
$$\approx \left(\frac{m_e T}{2\pi\hbar^2} \right)^{3/2} e^{-B/T} \quad (2)$$

key energy scale: H binding energy $B \equiv m_e + m_p - m_{\mathrm{H}} = 13.6$ eV

Q: why isn't recombination $X_e \to 0$ at $T_{\mathrm{rec}} \sim B$?



$T_1 = B_{\rm H}/10$
$T_2 = B_{\rm H}/5$
$T_3 = B_{\rm H}/3$
can ionize H
photon number fraction at ν: $\nu\, n_\nu$
frequency $h\nu/B_{\rm H}$
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



photon number per baryon: $\nu \, n_\nu / n_B$
frequency $h\nu / B_H$
can ionize H
$T_1 = B_H/3$
$T_2 = B_H/10$
$T_3 = B_H/30$
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Recombination "Delay"

Why is $T_{\text{rec}} \ll B$?
▷ because for small X_e, Saha says $X_e \propto 1/\eta^{1/2} \gg 1$
▷ many photons per baryon: even if typically $E_\gamma \ll B$,
high-E tail of Planck distribution not negligible (at first)
lots of ionizing photons with $E_\gamma \geq B$
H dissociated as soon as formed

When does dissociation stop?
can show that fraction of photons with $E_\gamma > B$
is roughly $f_{\text{ionizing}} \sim e^{-B/T}$
so ratio of ionizing photons per baryon is

$$\frac{n_{\gamma,\text{ionizing}}}{n_B} \sim \frac{e^{-B/T}}{\eta} \tag{3}$$

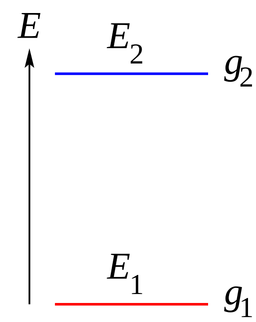
estimate recombination when $n_{\gamma,\text{ionizing}}/n_B \sim 1$
$\rightarrow T \sim B/\ln \eta^{-1} \ll B$ (check!)
$\Rightarrow$ recombination "delayed" to huge photon-to-baryon ratio

Recombination: Hydrogen Level Population

Boltzmann: consider a particle (elementary or composite)
with a series of energy states:

for two sets of states with energies E_1 and $E_2 > E_1$
and degeneracies (# states at each E) g_1 and g_2
ratio of number of particles in these states is

$$\frac{n(E_2)}{n(E_1)} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/T}$$



where I put $k = 1$, i.e., $kT \to T$

Example: atomic hydrogen, at T

Q: ratio of ground ($1S$) *to 1st excited state* ($2P$) *populations?*

apply to atomic hydrogen (H I):

Atomic hydrogen (H I):

- energy levels: $E_n = -B_{\mathrm{H}}/n^2$ for $n \geq 1$
- angular momenta degeneracies: $g_\ell = 2\ell + 1$

$1S$: $n = 1 \rightarrow E(1S) = -B$; $\ell = 0 \rightarrow g(1S) = 1$

$2P$: $n = 2 \rightarrow E(2P) = -B/4$; $\ell = 1 \rightarrow g(2P) = 3$

$$\frac{n(2P)}{n(1S)} = 3e^{-3B/4T} = 3e^{-120{,}000\,\mathrm{K}/T} \tag{4}$$

Q: sanity checks–is this physically reasonable?

consider recombining $p + e \rightarrow H + \gamma$ throughout recomb:

Q: what is γ energy at emission?

Q: what happens to γ?

Q: implications?

Recombination: Nonequilibrium Effects

for $p + e \rightarrow \mathrm{H}(n = 1) + \gamma$:

- $E_\gamma = B_\mathrm{H}$ "Lyman limit"
- H atoms *absorption cross section huge* at this energy
  photon mean free path $\ell = 1/n_\mathrm{H}\sigma_\mathrm{abs}$ tiny
  universe optically thick to Lyman photons

$\Rightarrow$ quickly reionizes another H atom! *no net change!*

To overcome delay

- recombine to 1st excited state: $p + e \rightarrow \mathrm{H}(2p) + \gamma$
- single photon $\mathrm{H}(2p) \rightarrow H(1s) + \gamma$ Lyα transition
  also optically think, also no net progress
- *two-photon transition* $\mathrm{H}(2p) \rightarrow H(1s) + \gamma + \gamma$ can go
  but probability & rate smaller than for single photon
- eventually redshifting takes Lyman photons off resonance

net effect: *delays recombination relative to Saha*

Last Scattering: Photons Decouple from Matter

"recombination" a smooth transition in X_e, not instantaneous
`www: equilibrium` X_e `plot`
nevertheless, exponential drop in X_e around $z_{\rm rec}$

photons interact with gas via Thomson scattering: $\gamma e \to \gamma e$
rate per photon of scattering with e:

$$\Gamma_e(\gamma) = n_e \sigma v = n_e \sigma_T c = X_e n_{\rm b} \sigma_T c \quad (5)$$

drop in $X_e \to$ abrupt slowdown in scattering

as usual, competition between interaction and expansion
interactions "stop" when

$$\Gamma_e(\gamma) \lesssim H \quad (6)$$

and solving for $\Gamma_e(T) = H(T)$ gives last scattering :

$$z_{\rm ls} \sim 1100 \quad (7)$$

After last scattering:

- photons “decoupled” from gas
- but $X_e \neq 0$: some free e, p remain

  Q: what is X_e *as* $T \to 0$*? why?*

Freezing of Recombination

when typical photon has last scattering with e
still some residual ionization: i.e., some free e, p
can they recombine? yes!
do they recombine? yes, for a short while...then no!

Why? recombination rate per p: $\Gamma_{\text{rec},p} \sim n_e \sigma_{\text{rec}} v_{\text{therm}}$
with $\sigma_{\text{rec}} \sim (m_e/T)\sigma_{\text{T}}$ and $v_{\text{therm}} \sim \sqrt{T/m_e}$
recombination stops when $\Gamma_{\text{rec},p} \lesssim H$

after this: cooling does not reduce ionization
fixed value of $X_{e,\text{freeze}} \sim 10^{-4}$: "freeze-in of residual ionization"
at

$$z_{\text{ri}} \simeq 1000 \tag{8}$$

Q: cosmological implications of $X_{e,\text{freeze}} \neq 0$?

Recombination Timeline Summarized

The large drop in free electron density around $z \sim 1000$ leads to three distinct but related events:

(1) recombination U. **ionized $\rightarrow$ neutral**

$X_e \rightarrow X_{e,\mathrm{rec}} \sim 0.1$: $z_{\mathrm{rec}} \sim 1300$

...but photons still coupled to gas, and vice versa

(2) last scattering typical photons no longer interacts with e

U. **opaque $\rightarrow$ transparent**

$\Gamma_e(\gamma) \sim H$: $z_{\mathrm{ls}} \sim 1100$

...but gas still coupled to photons *Q: how can this be?*

$T_{\mathrm{gas}} = T_{e,p,\mathrm{H}} = T_\gamma$

(3) residual ionization freeze-in

free e and p diluted until "can't find each other"

But even still: photons scatter off residual ionization
e and thus p, H still exchange energy
with thermal photon bath: $T_{e,p,\mathrm{H}} = T_\gamma$ still!
when does this stop?

(4) gas decoupling

typical residual e no longer has photon interactions
gas decouples from photons
when? Thomson scattering rate *per* e: $\Gamma_e = n_\gamma \sigma_\mathrm{T} c \lesssim H$
at $z_{\mathrm{dec,gas}} \sim 500$
note: scatter rate *per* e=$\Gamma_e \gg \Gamma_\gamma$=*scatter rate per CMB photon*

Summary of CMB Highlights

CMB Observed

can make precision observations of spectrum, sky distribution thanks to sophisticated radio techniques and instruments

- CMB fantastically isotropic: $\delta T/T \sim few \times 10^{-5}$
- CMB exquisitely thermal

CMB Theory

detailed, precise calculations of recomb, last scattering, thanks well-known atomic physics

- isotropic CMB $\rightarrow$ U. was once very homogeneous
- Planckian CMB spectrum $\rightarrow$ U. was once thermalized
  $\rightarrow$ plasma hot, dense enough to equilibrate

CMB $\rightarrow$ demands hot big bang in FLRW universe!

Extrapolated current U to $t \sim 400,000$ yr
  and $z \sim 1000 \rightarrow$ **great success!**

Emboldens us to push earlier!

Coda: The CMB in a Helium-Only Universe

Preflight 4 Discussion: Alternate Universe

- like ours in all respects, except:
- early Universe puts *all baryons in helium*: no hydrogen!

Q: does this universe have a CMB at all?

Q: how does the alt-CMB frequency spectrum I_ν compare to ours?

Q: what's same/different about recombination and decoupling?