Astro 507

Lecture 24

March 23, 2020

Announcements:

- **Problem Set 4 due today**
- Office Hours (online):
  Instructor: Wed 3-4pm, Fri 3-4pm
  TA: Thu noon-1pm
- **Preflight 5 due next Friday**

Last time: isotropic CMB wrapup
thermal CMB demands a hot, dense early Universe: big bang!
theory and observation agree at $z \sim 1000$, $t \sim 400$ kyr

emboldens us to push back to earlier times

Primordial Nucleosynthesis

Prelude to Nucleosynthesis

Big Bang Nucleosynthesis (BBN) similarities with recombination:
unbound components $\rightarrow$ bound states

Q: what sets T scale for element (nuclei) synthesis?

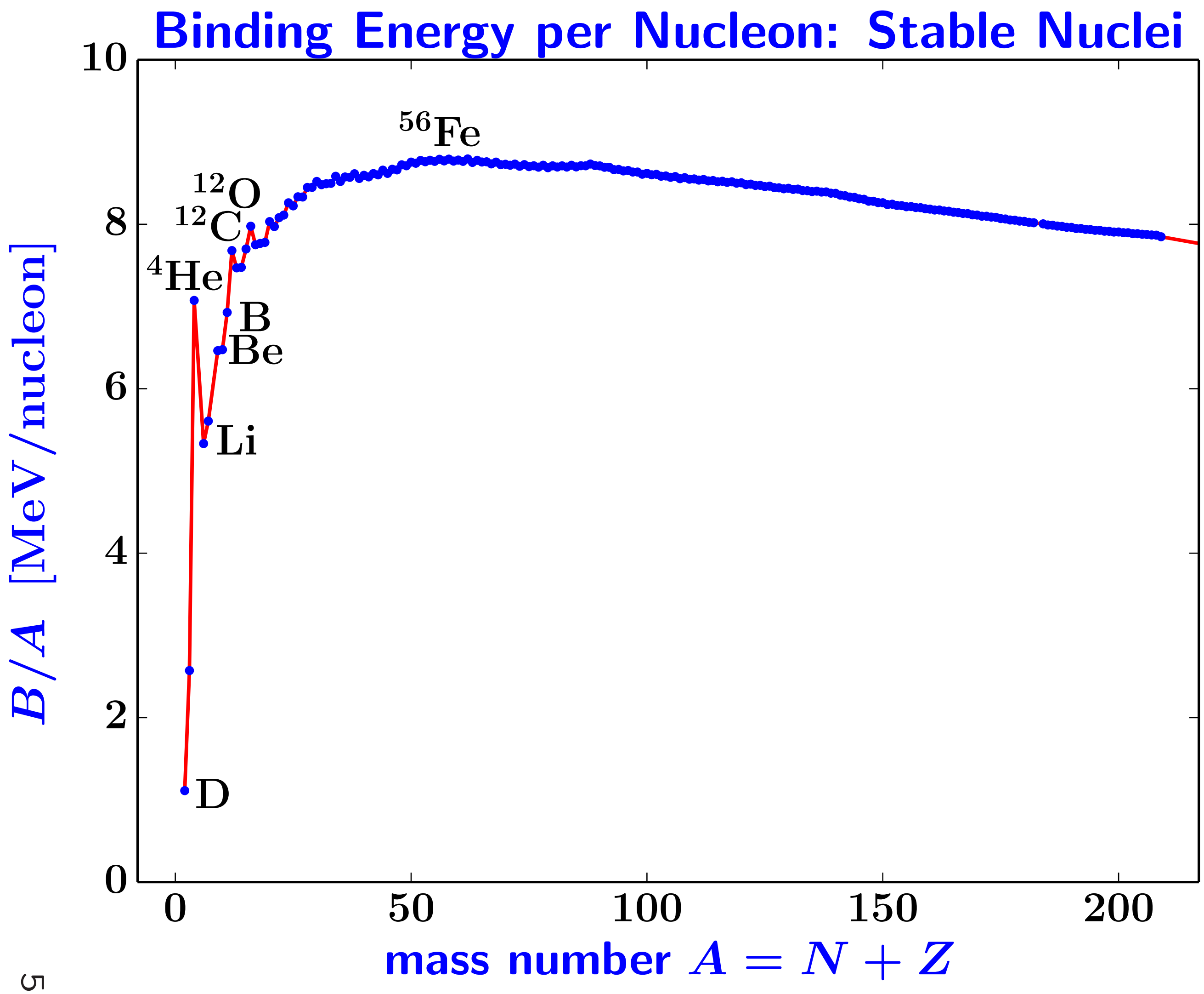
Q: what component dominates cosmic density, expansion then?

Q: what is the particle content of the universe then?

Nucleosynthesis: Nuclear Physics in a Nutshell

- nuclei are made of protons and neutrons: "**nucleons**"
- nucleon size ~ 1 fm $= 10^{-13}$ cm
- nucleon mass $m_p \approx m_n \approx 0.94$ GeV, but $m_n - m_p = 1.3$ MeV
  which means free neutrons are unstable, decay to protons
- nuclei are *quantum systems* bound by nuclear force, which is
  attractive at large distances $\gtrsim 1$ fm
  repulsive at shorter distances
- many nuclei exist with same proton number Z: "isotopes"

`www: chart of the nuclides--nuclear periodic table`



Q: what strikes you?

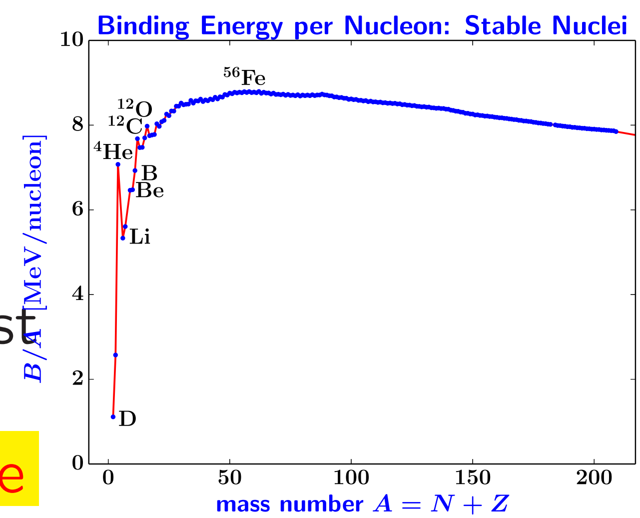
Binding Energy: Trends and Consequences

Overall nuclear binding energy features in Chart of Nuclides:

- highest binding along valley of stability
  $\Rightarrow$ stable isotopes are the most tightly bound

For stable nuclei:

- sharp rise in B_i/A_i at low A
- local max at ^{4}He
- *no stable nuclei at* $A = 5, 8$
- lightest stable nuclei include the lowest B/A for D, LiBeB
- *max B/A for middle masses:* **peak at** ^{56}Fe



Q: so what is rough energy scale for cosmic nucleosynthesis?

Nucleosynthesis: Setting the Stage

$\star$ nuclear binding energies typically $B \sim few$ MeV

$\star$ $T \sim$ MeV at redshift $z_{\rm bbn} = T/T_0 - 1 \sim 10^{10}$!
since $z_{\rm bbn} \gg z_{\rm eq} \sim 10^5$ (matter-rad equality)
well into radiation dominated era: $\rho \approx \rho_{\rm rad}$
`www:` Ω `vs` a `plot`
will see: $t(1 \text{ MeV}) \sim 1$ sec

$\star$ particle content at BBN
relativistic species: photons, neutrinos, $e^{\pm}$ when $T \gtrsim m_e$
non-relativistic species: baryons, e^- when $T \ll m_e$
what about dark matter? energy?
DM presumably non-rel, weakly interacting: inert during BBN
DE: also assume not important for dynamics, microphyiscs
...but can later relax these assumptions and test them!

Who Feels What? Particles and Forces

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{matrix} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \\ \end{matrix} \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{matrix} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \\ \end{matrix} \tag{1}$$

quarks: feel all fundamental forces (strong, EM, weak, gravity)
carry conserved quantum number: **baryon number**

leptons: do *not* feel strong force
but also carry conserved quantum number: **lepton number**

- *charged* leptons: feel EM, weak, gravity
- neutrinos: only feel weak, gravity

∞

More bragging rights:
in BBN, *all four* fundamental forces play a crucial role!

Neutrinos: Essential Ingredient yet Barely There

antineutrinos: $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$
since electric charge $Q(\nu) = 0$, possible that ν is own antiparticle
Q: is it?

masses: known that m_ν are nonzero (oscillations observed)
mass values not known (but for sure $\lesssim few \times 10$ eV $\ll m_e$)
Q: implications for BBN?
for quarks and charged leptons, masses increase with each family
$\rightarrow$ same for νs??

weak interaction: qualitative characteristics
(1) "signature" is transformation of quarks
e.g., β decays like $n \rightarrow p + e^- + \bar{\nu}_e$
really a quark change $d(ud) \rightarrow u(ud) + e^- + \bar{\nu}_e$
(2) for $E \lesssim 100$ GeV $(= M_W, M_Z)$, rxn strength is weak (duh!)
e.g., $\nu_e e \rightarrow \nu_e e$ scattering ~ 1 MeV: $\sigma_{\nu_e e} \sim 10^{-44}$ cm^2 $\sim 10^{-20} \sigma_T$

Nucleosynthesis: Particle Content Revisited

relativistic species:
γ, $\nu_i\bar{\nu}_i$ ($i \in e\mu\tau$), $e^\pm$ (for $T \gtrsim m_e$)

non-relativistic species:
baryons in BBN: when $T \gtrsim$ MeV: p, n only
when $T \leq m_e \rightarrow e$ non-relativistic too

⋆ neutrinos in BBN
Q: what sets n_ν, ρ_ν, T_ν*? how do they evolve?*
Q: assumptions needed?

BBN Initial Conditions: Ingredients of Primordial Soup

Begin above nuke binding: $\boxed{T > 1\ \mathrm{MeV}}$

EM reactions fast: typical rate $\Gamma_{\mathrm{EM}} \sim n_\gamma \sigma_{\mathrm{T}} c \gg H$
$\Rightarrow$ baryon, photon, $e^\pm$ pair plasma in thermal equilibrium:
$T_B = T_e = T_\gamma \equiv T$

Weak interaction fast too (for now)! $\Gamma_{\mathrm{weak}} \sim n_\nu \sigma_{\mathrm{weak}} c \gg H$
all ν species coupled to each other, and plasma
$\rightarrow T_\nu = T_\gamma$

For experts: What sets densities n_ν, ρ_ν?
not only T_ν, but also dreaded chem potential μ_ν
physics issue: is there a net neutrino excess: $n_\nu \neq n_{\bar{\nu}}$?
c.f. net baryon excess $\rightarrow$ exists: $n_B \neq n_{\bar{B}}$, but small: $n_B/n_\gamma \ll 1$
if net lepton number $n_L \sim n_B$, turns out $\mu_\nu/T \sim \eta$ negligible
we will assume $\mu_\nu \ll T \Leftrightarrow$ no large lepton/baryon excess
if otherwise, changes story!

BBN Initial Conditions: Radiation Domination

Neutrino densities: for sure $m_\nu \ll T$
assume $\mu_\nu \ll T$ $\rightarrow$ absolute n_ν, ρ_ν, P_ν set by T_ν
$\rightarrow$ each ν species i has $n_{\nu_i} = n_{\bar{\nu}_i}$ and

$$n_{\nu\bar{\nu},i} \propto T^3 = \frac{3}{4} n_\gamma \quad \rho_{\nu\bar{\nu},i} \propto T^4 = \frac{7}{8} \rho_\gamma \tag{2}$$

total relativistic energy density:

$$\rho_{\rm rel} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_{1\nu\bar{\nu}} \equiv g_* \frac{\pi^2}{30} T^4 \tag{3}$$

where g_* counts *"effective # of relativistic degrees of freedom"*
at $T \gtrsim 1$ MeV, $g_* = 43/4 = 10.75$, and Friedmann:

$$\frac{t}{1 \text{ sec}} \approx \left(\frac{1 \text{ MeV}}{T} \right)^2 \tag{4}$$

Q: simple way to see $t \sim 1/T^2$ *scaling is right?*

now focus on baryons *Q: what sets* n_B*?* n/p*?*

BBN Initial Conditions: The Baryons

baryon number: $B = \sum \text{baryons} - \sum \text{antibaryons}$
conserved at low energies
i.e., unchanged by reactions up to $E_{\rm LHC} \sim 10$ TeV $= 10^7$ MeV

So cosmic baryon density n_B not changed by reactions in BBN
▷ rather, set somehow in early universe ("cosmic baryogenesis")
▷ don't *a priori* know n_B, treat as free parameter (η)

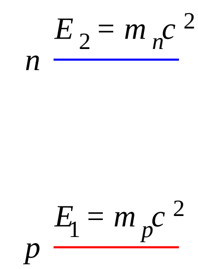
neutron-to-proton ratio n/p *can* and *does* change at ~ 1 MeV
weak int fast: $n \leftrightarrow p$ interconversion

$$n + \nu_e \leftrightarrow p + e^- \tag{5}$$
$$p + \bar{\nu}_e \leftrightarrow n + e^+ \tag{6}$$

also recall $m_n - m_p = 1.29$ MeV: close in mass but not same!

Q: implications for n/p?

n/p ratio "thermal"
think of as *2-state* system: the "nucleon,"



- nucleon "ground state" is the *proton*: $E_1 = m_p c^2$
- nucleon "excited state" is the *neutron*: $E_2 = m_n c^2$

when in equilibrium, Boltzmann sez:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \tag{7}$$

with $\Delta m = m_n - m_p = 1.293318 \pm 0.000009$ MeV

at $T \gg \Delta m$: $n/p \simeq 1$
at $T \ll \Delta m$: $n/p \simeq 0$

Equilibrium maintained until weak interactions freeze out
i.e., competition between weak physics, gravity physics
Q: how will weak freezeout scale compare to nuclear binding energy scale ~ 1 MeV?

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\text{weak}}$, i.e.,

$$\sqrt{G_{\text{N}}}T^2 \sim \sigma_0 m_e^{-2} T^5 \tag{8}$$

$$\Rightarrow T_{\text{weak freeze}} \sim \frac{(G_{\text{N}})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim \boxed{1 \text{ MeV}} \tag{9}$$

gravity & weak interactions conspire to give $T_{\text{f}} \sim m_e \sim B_{\text{nuke}}$!

for experts: note that $G_{\text{N}} = 1/M_{\text{Planck}}^2$, so

$$\frac{T^2}{M_{\text{Pl}}} \sim \alpha_{\text{weak}} \frac{T^5}{M_W^2} \tag{10}$$

$$\Rightarrow T_{\text{freeze}} \sim \left(\frac{M_W}{M_{\text{Pl}}}\right)^{1/3} M_W \sim 1 \text{ MeV} \tag{11}$$

freeze at nuclear scale, but by accident!

Q: what happens to n, p then? what else is going on?

Element Synthesis

first step in building complex nuclei: $n + p \to d + \gamma$
but $d + \gamma \to n + p$ until $T \ll B(d)$; see Extras

when photodissocation ineffective, $n + p \to d + \gamma$ fast
rapidly consumes all free n and builds d
which can be further processed to mass-3:

$$d + p \to {}^3\text{He} + \gamma \quad d + d \to {}^3\text{H} + p \quad d + d \to {}^3\text{He} + n \tag{12}$$

and to ^{4}He

$$^3\text{H} + d \to {}^4\text{He} + n \quad {}^3\text{He} + d \to {}^4\text{He} + p \tag{13}$$

some of which can then make mass-7:

$$^3\text{H} + {}^4\text{He} \to {}^7\text{Li} + \gamma \quad {}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma \tag{14}$$

Q: what limits how long these reactions can occur?
Q: which determines which products are most abundant?

BBN Reaction Flows

Binding Energy

nuclei are bound quantum structures, confined by nuclear forces among the "nucleons" n, p

can quantify degree of stability–i.e., resistance to destruction via binding energy: for nucleus with Z protons, N neutrons, $A = N + Z$ nucleons

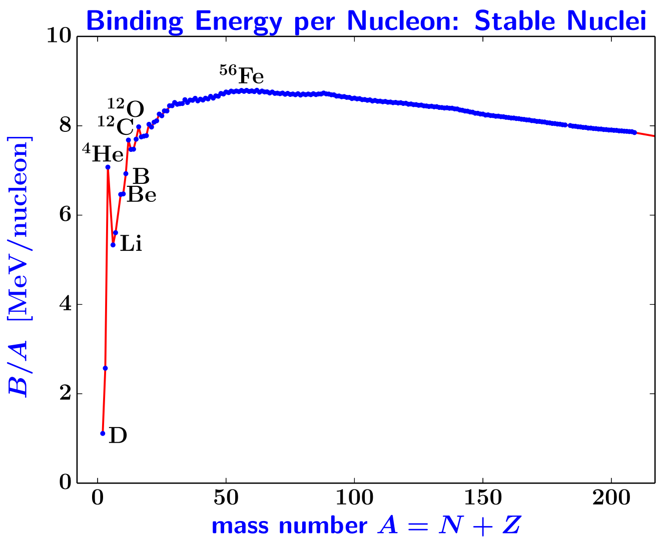
$$
\begin{aligned}
B_A &= \text{energy of individual parts} - \text{energy of bound whole} \\
&= (Zm_p + Nm_n - m_A)c^2 \\
&> 0 \quad \text{if bound}
\end{aligned}
$$

note: generally B_A increases with A

but that's not the whole story on stability

binding shared among all A nucleons,
so binding per nucleon is B_A/A

nuclear stability $\leftrightarrow$ high B_A/A



Q: implications for BBN

Reaction flows: tightest binding favored

$\rightarrow$ essentially all pathways flow to ^{4}He

`www: nuke network`

almost all $n \rightarrow {}^4$He:

$n(^4\text{He})_{\text{after}} = 1/2 \; n(n)_{\text{before}}$

$$Y_p = \frac{\rho(^4\text{He})}{\rho_B} \simeq 2(X_n)_{\text{before}} \simeq 0.24 \tag{15}$$

$\Rightarrow \sim 1/4$ of baryons into ^{4}He, 3/4 $p \rightarrow$H

result weakly (log) dependent on η

Robust prediction: large universal ^{4}He *abundance*

But $n \rightarrow {}^4$He incomplete: as nuke rxns freeze,
leave traces of:

- D
- ^{3}He (and ^{3}H$\rightarrow$^{3}He)
- ^{7}Li (and ^{7}Be$\rightarrow$^{7}Li)

abundances $\leftrightarrow$ nuke freeze T
trace species D, ^{3}He, ^{7}Li: strong $n_B \propto \eta$ dependence

BBN theory predictions summarized in **"Schramm Plot"**
Lite Elt Abundances vs η
`www: Schramm plot`

Note: no $A > 7$...so no C,O,Fe... *Q: why not?*

Why no elements $A > 7$?

1. Coulomb barrier

2. nuclear physics: "mass gaps"
no stable nuclei have masses $A = 5, 8$
$\rightarrow$ with just p & ^{4}He, can't overcome via 2-body rxs
need 3-body rxns (e.g., $3\alpha \rightarrow {}^{12}$C) to jump gaps
but ρ, T too low

Stars *do* jump this gap, but only because have higher density a long time compared to BBN

Testing BBN: Warmup

BBN Predictions: Lite Elements vs η

To test: measure abundances

Where and when do BBN abundances (Schramm plot) apply?

Look around the room–not 76% H, 24% He.
Is this a problem? Why not?

Solar system has metals not predicted by BBN
Is this a problem? Why not?

So how test BBN? What is the key issue?

When does first non-BBN processing start?

Testing BBN: Lite Elements Observed

Prediction:

BBN Theory $\rightarrow$ lite elements at $t \sim 3$ min, $z \sim 10^9$

Problem:

observe lite elements in astrophysical settings

typically $t \gtrsim 1$ Gyr, $z \lesssim few$

stellar processing alters abundances

Q: If measure abundances in a real astrophysical system, can you unambiguously tell that stars have polluted?

Q: How can we minimize (and measure) pollution level?

stars not only alter light elements

but also make heavy element = "metals"

stellar cycling: metals $\leftrightarrow$ time

Solution:

$\rightarrow$ measure lite elts and metals

low metallicity $\rightarrow$ more primitive

in limit of metals $\rightarrow$ 0: primordial abundances!

look for regions with low metallicity $\rightarrow$ less processing

Directors' Cut Extras

Elementary Particles for the Minimalist
Antimatter

fundamental result of Relativistic QM
every particle has an antiparticle
e.g., $\overline{e^-} = e^+$ positron
e.g., $\bar{p} =$ antiproton; Fermilab: $p\bar{p}$ collisions

note: mass $m(\bar{x}) = m(x)$
decay lifetime $\tau(\bar{x}) = \tau(x)$
spin $S(\bar{x}) = S(x)$
electric charge $Q(\bar{x}) = -Q(x)$

sometimes particle = own antiparticle (must have charge 0)
e.g., $\bar{\gamma} = \gamma$, but note: $\bar{n} \neq n$

Cosmic Antimatter: rule of thumb
$x, \bar{x}$ abundant when thermally produced: $T > m_x$

Baryons

n and p not fundamental particles
made of 3 pointlike particles: "quarks"
two types ("flavors") in n, p: u "up," d "down"
$p = uud$, $n = udd \to$ quark electric charge $Q_u = +2/3$, $Q_d = -1/3$
spin $S(u) = 1/2 = S(d)$

baryon $\equiv$ made of 3 quarks

baryon conservation:
assign "baryon number" $A(q) = +1/3$, $A(\bar{q}) = -1/3$
$\to A(n) = A(p) = +1$
in all known interactions: baryon number conserved:
$\sum A_{\text{init}} = \sum A_{\text{fin}}$
$\to$ guarantees stability of the proton *Q: why?*
but free n unstable, decay to p *Q: why not n decay in nuclei?*

Periodic Table of Elementary Particles

known fundamental particles (& antipartners): 3 **families**

$$
\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix}
\begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix}
\begin{array}{l} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \ \end{array}
\begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}
\begin{array}{l} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \ \end{array}
\tag{16}
$$

all of these are spin-1/2: matter made of fermions!

Family Resemblances

1st family: quarks, charged lepton (e) comprise ordinary matter

2nd, 3rd family particles

- same electric charges, same spins, (mostly) same interactions as corresponding 1st family cousins
- but 2nd, 3rd family quarks, charged leptons more massive and & *unstable* $\rightarrow$ decay into 1st family cousins

lifetimes very short, e.g., longest is $\tau(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = 2 \times 10^{-6}$ s

Q: implications for BBN epoch?

Weak $n \leftrightarrow p$ Rates

example: want rate Γ_n per n of $\nu + n \rightarrow e^- + p$
as func. of T

Generally,

$$\Gamma_n = n_\nu \langle \sigma v \rangle \sim T^3 \langle \sigma \rangle \tag{17}$$

since $v_\nu \simeq c$

can show: cross section $\boxed{\sigma \sim \sigma_0 (E_e/m_e)^2}$
where $\sigma_0 \sim 10^{-44}$ cm^2 very small!
so thermal avg: $\langle \sigma \rangle \sim \sigma_0 (T/m_e)^2$

for experts: $\sigma \sim G_F^2 T^2 \sim \alpha_{\text{weak}} T^2 / M_W^4$
so $\Gamma_{\text{weak}} \sim \alpha_{\text{weak}} T^5 / M_W^4$