Astro 507

Lecture 25

March 30, 2020

Announcements:

- **Preflight 5: due Friday**

Last time: began big-bang nucleosynthesis (BBN)

Q: BBN vs CMB similarities? differences?

Q: characteristic T?

Q: what dominates cosmic expansion?

Q: density when all particles have same T?

BBN Initial Conditions: Radiation Domination

Neutrino densities: relativistic (non-degenerate) thermal fermions
densities set by T and fundamental consts

$$n_{\nu\bar{\nu},i} \propto T^3 = \frac{3}{4} n_\gamma \quad \rho_{\nu\bar{\nu},i} \propto T^4 = \frac{7}{8} \rho_\gamma \tag{1}$$

total relativistic energy density:

$$\rho_{\rm rel} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_{1\nu\bar{\nu}} \equiv g_* \frac{\pi^2}{30} T^4 \tag{2}$$

where g_* counts *"effective # of relativistic degrees of freedom"*
at $T \gtrsim 1$ MeV, $g_* = 43/4 = 10.75$, and Friedmann:

$$\frac{t}{1 \text{ sec}} \approx \left(\frac{1 \text{ MeV}}{T} \right)^2 \tag{3}$$

Q: simple way to see $t \sim 1/T^2$ scaling is right?

now focus on baryons *Q: what sets n_B? n/p?*

BBN Initial Conditions: The Baryons

baryon number: $B = \sum \text{baryons} - \sum \text{antibaryons}$
conserved at low energies
i.e., unchanged by reactions up to $E_{\text{LHC}} \sim 10$ TeV $= 10^7$ MeV

So cosmic baryon density n_B not changed by reactions in BBN
▷ rather, set somehow in early universe ("cosmic baryogenesis")
▷ don't *a priori* know n_B, treat as free parameter (η)

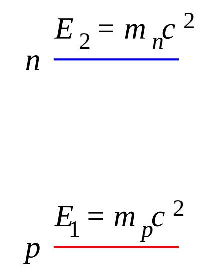
neutron-to-proton ratio n/p *can* and *does* change at ~ 1 MeV
weak interaction fast: rapid $n \leftrightarrow p$ interconversion

$$n + \nu_e \leftrightarrow p + e^- \tag{4}$$
$$p + \bar{\nu}_e \leftrightarrow n + e^+ \tag{5}$$

also recall $m_n - m_p = 1.29$ MeV: close in mass but not same!

Q: implications for n/p?

n/p ratio "thermal"
think of as *2-state* system: the "nucleon,"



- nucleon "ground state" is the *proton*: $E_1 = m_p c^2$
- nucleon "excited state" is the *neutron*: $E_2 = m_n c^2$

when in equilibrium, Boltzmann sez:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \tag{6}$$

with $\Delta m = m_n - m_p = 1.293318 \pm 0.000009$ MeV

at $T \gg \Delta m$: $n/p \simeq 1$
at $T \ll \Delta m$: $n/p \simeq 0$

Equilibrium maintained until weak interactions freeze out
i.e., competition between weak physics, gravity physics
Q: how will weak freezeout scale compare to nuclear binding energy scale ~ 1 *MeV?*

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\rm weak}$, i.e.,

$$\sqrt{G_{\rm N}} T^2 \ \sim \ \sigma_0 m_e^{-2} T^5 \tag{7}$$

$$\Rightarrow T_{\rm weak\ freeze} \ \sim \ \frac{(G_{\rm N})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim \boxed{1\ \rm MeV} \tag{8}$$

gravity & weak interactions conspire to give $T_{\rm f} \sim m_e \sim B_{\rm nuke}$!

for experts: note that $G_{\rm N} = 1/M_{\rm Planck}^2$, so

$$\frac{T^2}{M_{\rm Pl}} \ \sim \ \alpha_{\rm weak} \frac{T^5}{M_W^2} \tag{9}$$

$$\Rightarrow T_{\rm freeze} \ \sim \ \left(\frac{M_W}{M_{\rm Pl}}\right)^{1/3} M_W \sim 1\ \rm MeV \tag{10}$$

freeze at nuclear scale, but by accident!

Q: what happens to n, p *then? what else is going on?*

Element Synthesis

first step in building complex nuclei: $n + p \rightarrow d + \gamma$
but $d + \gamma \rightarrow n + p$ until $T \ll B(d)$; see Extras

when photodissocation ineffective, $n + p \rightarrow d + \gamma$ fast
rapidly consumes all free n and builds d
which can be further processed to mass-3:

$$d + p \rightarrow {}^3\mathrm{He} + \gamma \quad d + d \rightarrow {}^3\mathrm{H} + p \quad d + d \rightarrow {}^3\mathrm{He} + n \tag{11}$$

and to ^{4}He

$$ {}^3\mathrm{H} + d \rightarrow {}^4\mathrm{He} + n \quad {}^3\mathrm{He} + d \rightarrow {}^4\mathrm{He} + p \tag{12}$$

some of which can then make mass-7:

$$ {}^3\mathrm{H} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Li} + \gamma \quad {}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma \tag{13}$$

Q: what limits how long these reactions can occur?
Q: which determines which products are most abundant?

BBN Reaction Flows

Binding Energy

nuclei are bound quantum structures, confined by nuclear forces among the "nucleons" n, p

can quantify degree of stability–i.e., resistance to destruction via binding energy: for nucleus with Z protons, N neutrons, $A = N + Z$ nucleons

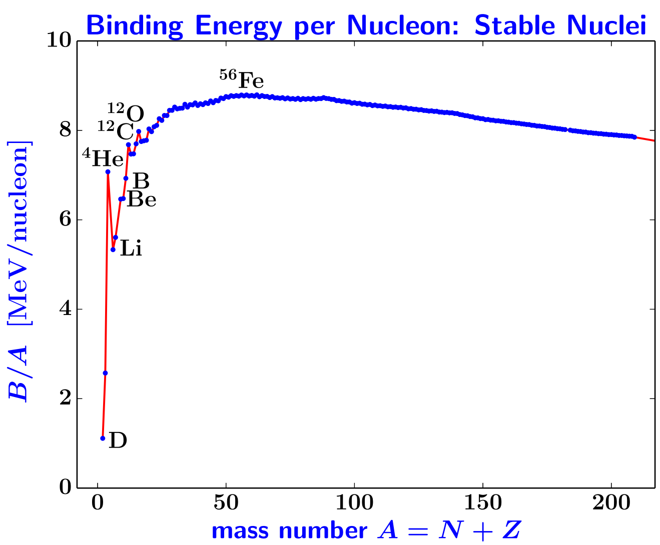
$$\begin{aligned} B_A &= \text{energy of individual parts} - \text{energy of bound whole} \\ &= (Z m_p + N m_n - m_A)c^2 \\ &> 0 \ \text{ if bound} \end{aligned}$$

note: generally B_A increases with A

but that's not the whole story on stability

binding shared among all A nucleons,
so binding per nucleon is B_A/A

nuclear stability $\leftrightarrow$ high B_A/A



Q: implications for BBN

Reaction flows: tightest binding favored

$\rightarrow$ essentially all pathways flow to ^{4}He

`www: nuke network`

almost all $n \rightarrow ^4$He:

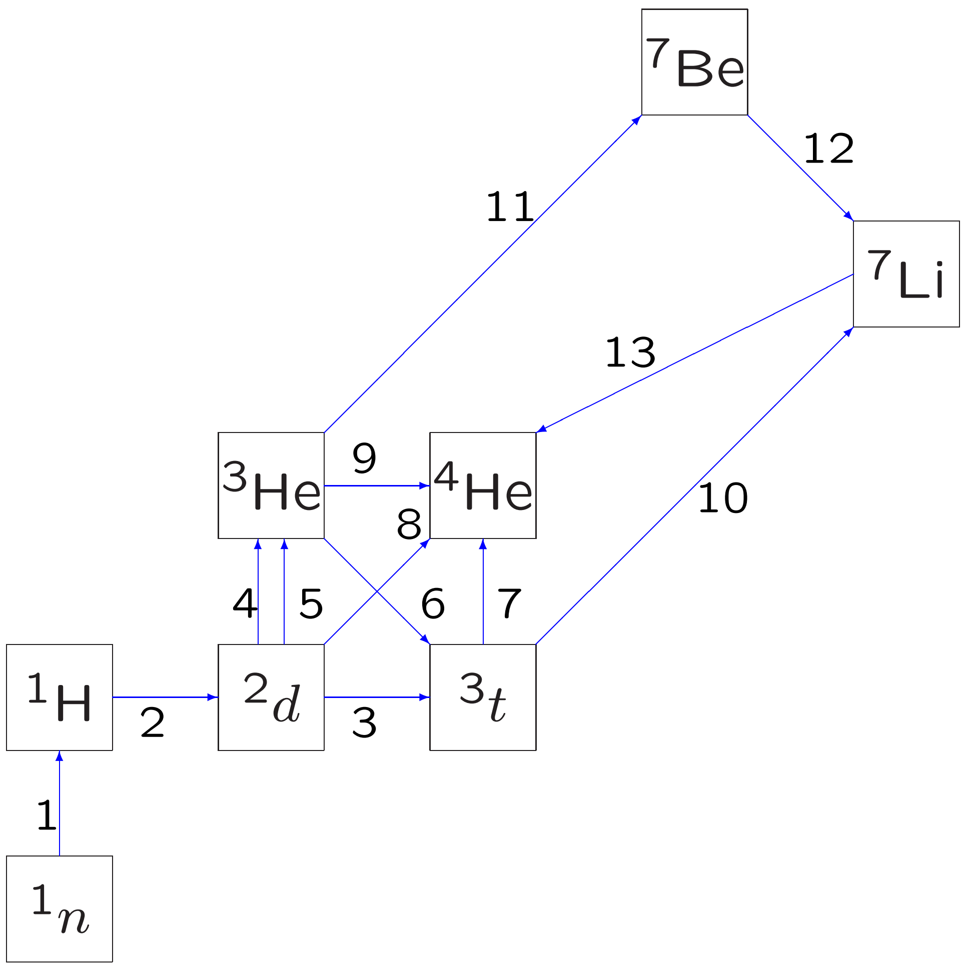
$n(^4\text{He})_{\text{after}} = 1/2 \; n(n)_{\text{before}}$

$$Y_p = \frac{\rho(^4\text{He})}{\rho_B} \simeq 2(X_n)_{\text{before}} \simeq 0.24 \tag{14}$$

$\Rightarrow \sim 1/4$ of baryons into ^{4}He, $3/4$ $p \rightarrow$H

result weakly (log) dependent on η

Robust prediction: large universal ^{4}He abundance



1: $n \to p\,e\,\nu$
2: $n(p,\gamma)d$
3: $d(d,p)t$
4: $d(p,\gamma)^3$He
5: $d(d,n)^3$He
6: ^{3}He$(n,p)t$
7: $t(d,n)^4$He
8: $d(d,\gamma)^4$He
9: ^{3}He$(d,p)^4$He
10: $t(\alpha,\gamma)^7$Li
11: ^{4}He$(\alpha,\gamma)^7$Be
12: ^{7}Be$(n,p)^7$Li
13: ^{7}Li$(p,\alpha)^4$He

But $n \rightarrow {}^4$He incomplete: as nuke rxns freeze,
leave traces of:

- D
- ^{3}He (and ^{3}H$\rightarrow$^{3}He)
- ^{7}Li (and ^{7}Be$\rightarrow$^{7}Li)

abundances $\leftrightarrow$ nuke freeze T
trace species D, ^{3}He, ^{7}Li: strong $n_B \propto \eta$ dependence

BBN theory predictions summarized in **"Schramm Plot"**

Lite Elt Abundances vs η

`www: Schramm plot`

Note: no $A > 7$...so no C,O,Fe... *Q: why not?*

Why no elements $A > 7$?

1. Coulomb barrier

2. nuclear physics: "mass gaps"
no stable nuclei have masses $A = 5, 8$
$\rightarrow$ with just p & ^{4}He, can't overcome via 2-body rxs
need 3-body rxns (e.g., $3\alpha \rightarrow ^{12}$C) to jump gaps
but ρ, T too low

Stars *do* jump this gap, but only because have higher density a long time compared to BBN

Testing BBN: Warmup

BBN Predictions: Lite Elements vs η

To test: measure abundances

Where and when do BBN abundances (Schramm plot) apply?

Look around the room–not 76% H, 24% He.
Is this a problem? Why not?

Solar system has metals not predicted by BBN
Is this a problem? Why not?

So how test BBN? What is the key issue?

When does first non-BBN processing start?

Testing BBN: Lite Elements Observed

Prediction:

BBN Theory $\rightarrow$ lite elements at $t \sim 3$ min, $z \sim 10^9$

Problem:

observe lite elements in astrophysical settings

typically $t \gtrsim 1$ Gyr, $z \lesssim few$

stellar processing alters abundances

Q: If measure abundances in a real astrophysical system, can you unambiguously tell that stars have polluted?

Q: How can we minimize (and measure) pollution level?

stars not only alter light elements

but also make heavy element = “metals”

stellar cycling: metals $\leftrightarrow$ time

Solution:

$\rightarrow$ measure lite elts and metals

low metallicity $\rightarrow$ more primitive

in limit of metals $\rightarrow$ 0: primordial abundances!

look for regions with low metallicity $\rightarrow$ less processing

Deuterium

Two methods:

(1) use $D/H_{\odot}$, model $D-Z$ evolution:
model dependent **X** (old school)

(2) measure D/H at high z **YES**
"quasar absorption line systems"

QSO: for our purposes
high-z continuum source (lightbulb)

`www: QSO spectrum`

consider cloud, mostly H

- at $z < z_{\rm qso}$, but still high z

  e.g., $z_{\rm qso} = 3.4, z_{\rm cloud} = 3$
- H absorbs γ if energy tuned to levels

  lowest: $n = 1 \to 2$, Lyα
- but Lyα in QSO frame

  redshifted in cloud frame

What happens?

What about a cloud at yet lower z?

intervening material seen via absorption

H: "Lyman-α forest"

Deuterium in High-z Absorption Systems

D energy levels $\neq$ H: for Hydrogen-like atoms

$$E_n = -\frac{1}{n^2}\,\frac{1}{2}\,\alpha^2 \mu c^2 \tag{15}$$

where μ = **reduced mass** $= m_e m_A/(m_e + m_A) \simeq m_e(1 - m_e/Am_p)$

$\Rightarrow \Delta E = E_{n,D} - E_{n,H} \approx +1/2\ m_e/m_p\ E_{n,H}$

$\Rightarrow \Delta z_D = \Delta\lambda/\lambda = -1/2\ m_e/m_p$

$c\Delta z_D = -82$ km/s (blueward) $\rightarrow$ look for "thumbprint"

`www: O'Meara D spectrum`

What about stellar processing?

★ stars *destroy* D *before* H-burning! (pre-MS)

★ nonstellar astrophysical (Galactic) sources negligible

Epstein, Lattimer & Schramm 1977; updated in Prodanović & BDF 03)

$\Rightarrow$ **BBN is only important D nucleosynthesis source**

$\rightarrow$ *$D(t)$ only decreases*

chem evol models: versus Z metallicity: $D \sim e^{-Z/Z_\odot} D_p$

Quasar absorbers: $Z \sim 10^{-2} Z_\odot \rightarrow$ expect $D_{\mathrm{QSOALS}} \approx D_p$

Deuterium Results

Until recently: the 7 best systems
(clean D, well-determined H)

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{QSOALS}} = \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_p = (2.78 \pm 0.29) \times 10^{-5} \tag{16}$$

Cooke, Pettini (2012, 2013): new very high-precision systems
Damped Lyα absorbers (DLAs):

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{QSOALS}} = \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_p = (2.53 \pm 0.04) \times 10^{-5} \tag{17}$$

now a 2% measurement!

Directors' Cut Extras

The Short but Interesting Life of a Neutron

(1) at $T > T_f$, $t \sim 1$ s
$n \leftrightarrow p$ rapid
maintain $n/p = e^{-\Delta m/T}$

(2) at $T = T_f$,
fix $n/p = e^{-\Delta m/T_f} \simeq 1/6$
so n "mass fraction" is

$$X_n = \frac{\rho_n}{\rho_B} = \frac{m_n n}{m_n n + m_p p} \approx \frac{n}{n+p} \approx 1/7 \tag{18}$$

(3) until nuclei form,
free n decay: $\dot{n} = -n/\tau_n$, with $\tau_n = 885.7 \pm 0.8$ s
then mass fraction drops as

$$X_n = X_{n,i} e^{-\Delta t/\tau} \tag{19}$$

Q: why take this simple from?

Nuclear Astrophysics: Overcoming the Coulomb Barrier

to go from n, p to ^{4}He requires
at least one nuclear reactions between charged nuclei
so must contend with Coulomb repulsion

$$V_C(r) = \frac{Z_1 Z_2 e^2}{r} \sim 1 \ Z_1 Z_2 \ \text{MeV} \ \left(\frac{1 \ \text{fm}}{r}\right) \tag{20}$$

but nuclear force, while strong, is short-ranged: $r_{\text{nuke}} \sim 1$ fm
$\rightarrow$ particles apparently need $mv^2/2 \sim |V_C| \sim 1$ MeV to fuse
but $mv^2/2 \sim T \ll 1$ MeV, and higher energies exponentially suppressed

Q: how can we overcome this barrier?

Quantum Mechanics to the Rescue

Quantum mechanics $\rightarrow$ tunneling
Penetration probability

$$P \propto e^{-2\pi Z_1 Z_2 e^2/\hbar v} = e^{-bE^{-1/2}} \tag{21}$$

so $P \neq 0$ even when $E \ll |V_C|$
$\rightarrow$ tunnel under barrier, then react
note: not as serious an issue in BBN as it is in most stars
e.g., the sun