Astro 507

Lecture 30

April 10, 2020

Announcements:

- **Problem Set 5 due Monday**
  Revised (reduced) questions posted April 7
  can post questions in Homework Discussion
- **Preflight 6: Part (a) Due Friday April 17**
  Wikipedia Cosmology!

Last time: began cosmological inflation

⋆ highest z, earliest t we will visit

⋆ transition from homogeneous $\rightarrow$ inhomogeneous Universe

⋆ afterward, we will go forward in t
  study how inflationary (?) density perturbations
  are written onto CMB and grow to structures today

Q: what cosmic puzzles does inflation solve?

Q: solutions to these puzzles without inflation?

Q: how does inflation differ from usual cosmic expansion?

Inflation motivation: cosmic puzzles

flatness, horizon, monopoles, lumpiness

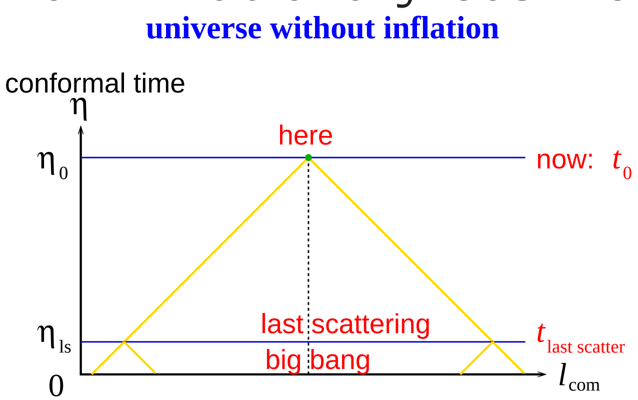
Inflation: early period of *rapid, accelerated expansion*

scale factor growth $a_{\text{post-inf}}/a_{\text{pre-inf}} \sim e^{60} \sim 10^{26}$

Simultaneously solves flatness, horizon, monopoles

flatness: acceleration drives $|\Omega - 1| \to 0$

non-inflationary cosmic spacetime:



Q: how does inflation change this?

spacetime plot is *conformal time* $\eta = \int_0^t dt/a(t)$

versus comoving distance $r_{\rm com}$

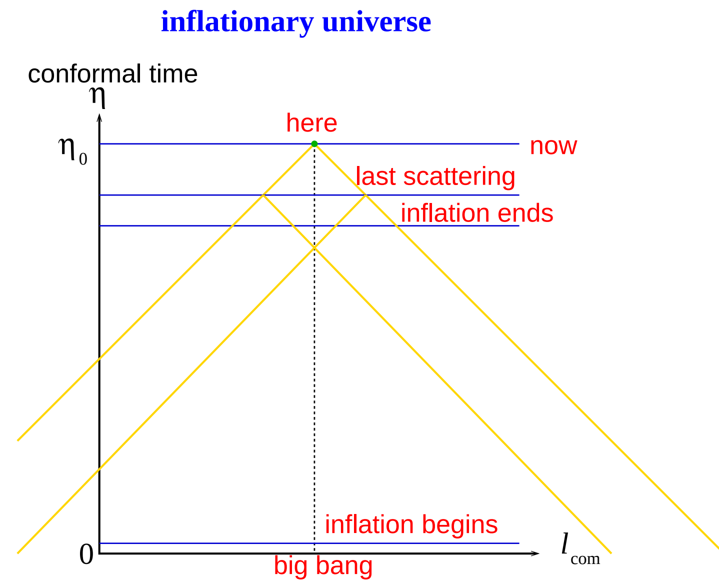
inflation: over tiny timeframe Δt,

scale factor $a(t)$ grows by $a_f/a_i \sim e^N$

with $N \approx 60$ "e-foldings"

growth during infation: $\sim$exponential

$$\delta\eta \approx \frac{1}{a_i}\int_0^{\Delta t}\frac{dt}{e^{Ht}} \sim e^N t_f$$



$\Rightarrow$ huge growth in conformal time during inflation

due to acceleration (exponentiation)

Q: but how to make early U accelerate?

Scalar Fields

Now introduce a new fundamental interaction ("fifth force")

classical field: a *scalar* $\phi(\vec{r}, t)$
a single-valued function at each point of spacetime

simplest case: ϕ only interacts with itself

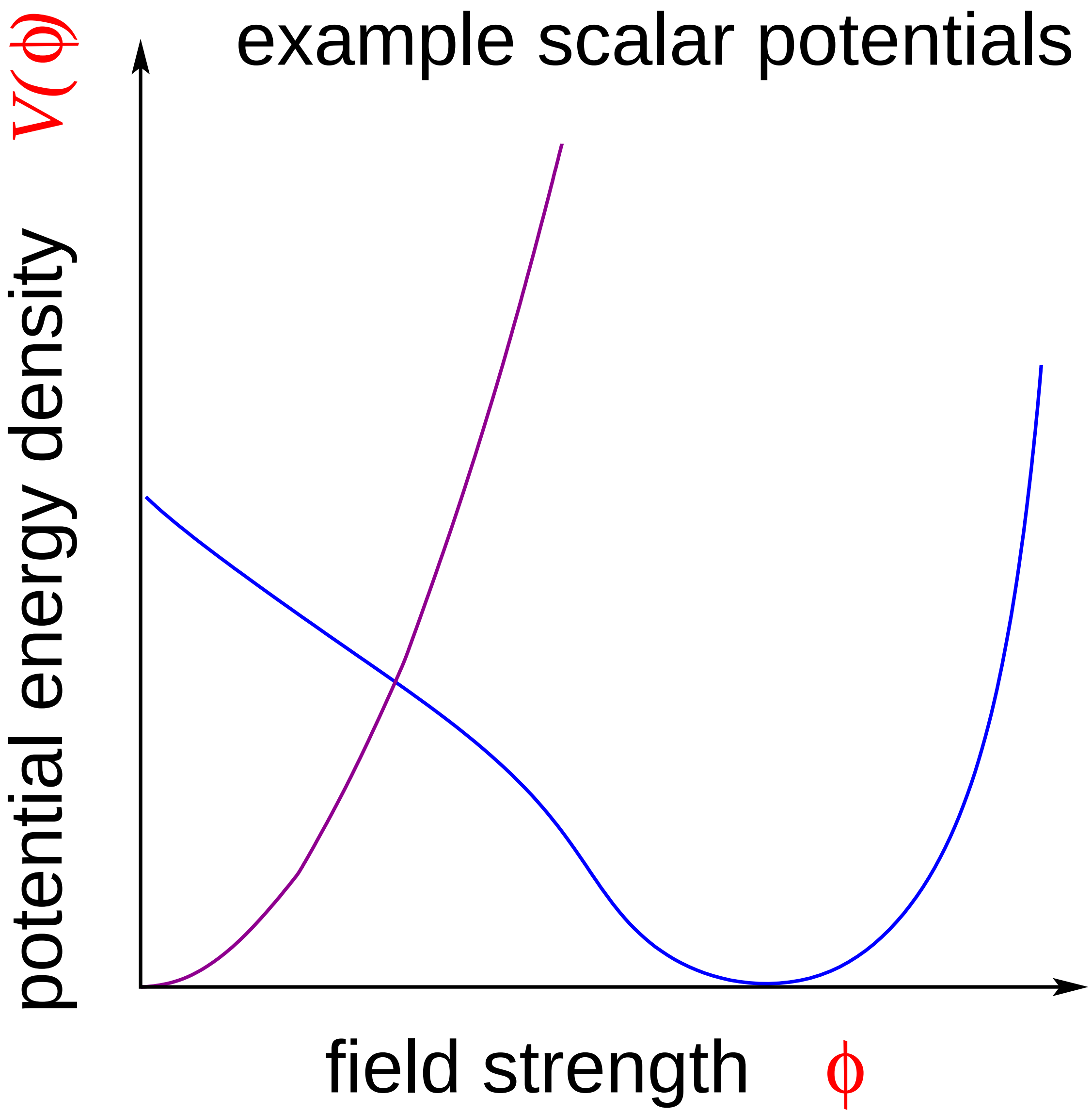
- scalar energy density

$$\varepsilon = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{1}$$

  kinetic term depends on time change
  potential describing self-interaction:
  examples: $V(\phi) = m^2\phi^2/2$, or $V = \lambda\phi^4$

- scalar pressure – note crucial sign flip

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{2}$$


example scalar potentials
potential energy density
$V(\phi)$
field strength
ϕ

Inflationary Cosmology

recall from Dark Energy discussion: acceleration demands $P < 0$
can't do this with matter or radiation

But:
★ *scalar field* ϕ can have $P_\phi < 0$
★ scalar fields *required* for electroweak unification
and appear in all other unification schemes

Alan Guth (1981)
if early Universe
▷ contains a *scalar field*,
▷ whose *potential energy dominates*: $\rho_\phi \approx V_\phi \approx \rho_{\text{tot}}$
then (in 21st century language) $w_\phi \rightarrow -1$
$\rightarrow$ *cosmic acceleration and exponential expansion!*

Cosmic Scalar Fields: Episode II

let cosmic scalar field ϕ be "minimally coupled" – i.e.,

- interacts only to itself via potential $V(\phi)$
- and gravity, via ρ_ϕ

Properties: Note $\hbar = c = 1!$ $\Rightarrow [\phi] = [E] = [\ell^{-1}] = [t^{-1}]$

Equation of motion $$\ddot{\phi} - \nabla^2\phi + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad (3)$$

energy density $$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V \qquad (4)$$

pressure $$P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 - V \qquad (5)$$

why? Lagrangian dens $\mathcal{L} = 1/2\ \partial_\mu\phi\partial^\mu\phi - V$ $\Rightarrow$ stress-energy

∞ $$T_{\mu\nu} \equiv \mathrm{diag}(\rho_\phi, p_\phi, p_\phi, p_\phi) \qquad (6)$$

$$= \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L} = (\partial_\mu\phi\partial_\nu\phi - \frac{g_{\mu\nu}}{2}\partial_\mu\phi\partial^\mu\phi) + g_{\mu\nu}V \qquad (7)$$

Scalar Field: Cosmic Equation of Motion

for homogeneous field $\phi(t,\vec{x}) = \phi(t)$, so

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V \quad (8)$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V \quad (9)$$

apply "first law of cosmo-thermodynamics" (useful for PS5)

$$\begin{aligned}
d(\rho a^3)/dt + p d(a^3)/dt &= (\rho + p) d(a^3)/dt + a^3 d\rho/dt = 0 \\
(\rho + p)\frac{d(a^3)/dt}{a^3} + \dot{\rho} &= 3H(\rho + p) + \dot{\rho} \\
&= 3H\dot{\phi}^2 + \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V\right) \\
&= 3H\dot{\phi}^2 + \dot{\phi}\ddot{\phi} + dV/d\phi\ \dot{\phi} = 0
\end{aligned}$$

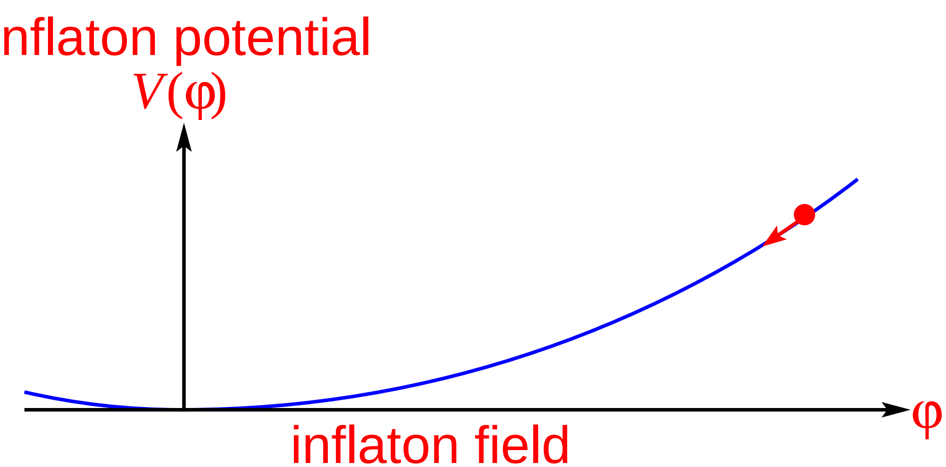
which gives $\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$

Scalar Field Time Evolution

so for homogeneous field $\phi(t,\vec{x}) = \phi(t)$

field equation of motion analogous to Maxwell is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$



formally: same as Newtonian ball rolling down hill V

but impeded by friction ("Hubble drag") $3H$

Scalar Fields and Cosmic Accelerants

pressure and energy density

$$\rho_\phi = \dot{\phi}^2/2 + V \quad (10)$$
$$P_\phi = \dot{\phi}^2/2 - V \quad (11)$$

which gives **equation of state** parameter

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \quad (12)$$

Q: limiting cases?

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \quad (13)$$

if kinetic term dominates: $\dot{\phi}^2 \gg V$

$w_\phi \to +1$: $P_\phi = \rho_\phi$, deceleration

if potential term dominates: $\dot{\phi}^2 \ll V$

$w_\phi \to -1$: $P_\phi = -\rho_\phi$, **acceleration!**

Note: same motivation for scalar field models of dark energy!

Q: requirements of workable inflation scenario?

Ingredients of an Inflationary Scenario

Recipe:

1. **inflaton field ϕ must exist** in early U.
2. must have $\rho_\phi \approx V$ so that $w_\phi \to -1$ so that $a \sim e^{Ht}$
3. continue to exponentiate $a \sim e^N a_{\rm init}$
   for at least $N = \int H\,dt \gtrsim 60$ e-folds
4. stop exponentiating eventually (**"graceful exit"**)
5. convert field ρ_ϕ back to radiation, matter (**"reheating"**)
6. then ϕ must "keep a low profile," $\rho_\phi \ll \rho_{\rm tot}$

7 (bonus) what about acceleration and dark energy today?
   is quintessence a rebirth of inflationary ϕ?
      goal of "quintessential inflation" models

Q: to meet 2: $\rho_\phi \approx V \rightarrow$ what does this mean?

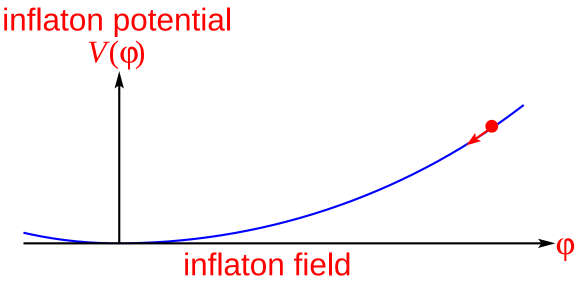
Intermission: Hall of Famers

To inflate, need slow ϕ evolution:

$\ddot{\phi} \ll 3H\dot{\phi}$ $\leftrightarrow$ friction large:

$\Rightarrow$ achieve "terminal speed"

$$\dot{\phi} \approx -\frac{1}{3H} V'$$



Slowness conditions $\dot{\phi}^2/2 \ll V$ and $\ddot{\phi} \ll 3H\dot{\phi}$ constrain **"slow-roll parameters"**:

$$\epsilon(\phi) = \frac{m_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad (14)$$

$$\eta(\phi) = m_{\rm pl}^2 \frac{V''}{V} \quad (15)$$

to be **small**: $\epsilon \ll 1$ and $|\eta| \ll 1$

(you'll get to show this in PS5)

Q: Why is it useful to express slow-roll criteria this way?

Q: What do these imply about the nature of V?

Q: What about magnitude of ϕ during inflation?

The Charms of a Slow Roll

Usefulness of slow roll parameters ϵ, η

★ ϵ, η quantify conditions for *maintaining* inflation
purely in terms of underlying potential V
$\rightarrow$ an immediate constraint on inflaton physics
i.e., any workable potential most satisfy slow roll
want derivatives small $\rightarrow$ need flat potential

★ ϵ, η quantify inflaton energy scale
• typically expect $V'/V \sim 1/\phi$
• but slow roll $(V'/V)^2 \sim \epsilon \; m_{\rm pl}^2$
together these give $\phi \gtrsim m_{\rm pl}$ during inflation

Hints for Problem Set 5 Q4

You may *assume that the slow roll condition holds*

$$\dot{\phi} \simeq -\frac{V'}{3H}$$

and assume $\rho_{\rm total} \approx \rho_\phi$: $H^2 \approx V/3m_{\rm pl}^2$

even so, not all potentials V give successful inflation

Q4(b):
show that $\dot{\phi}^2/2 \ll V$ requires $\epsilon \ll 1$

Q4(c):
show that $\ddot{\phi} \ll 3H\dot{\phi}$ requres $|\eta| \ll 1$ (and $\epsilon \ll 1$ too)

generically expect $\phi \gtrsim m_{\rm pl}$

$\Rightarrow$ for successful inflation, field probes the Planck scale (?)

;-) a good thing?

hints at quantum gravity

if $\Omega_{\rm init} \gtrsim 1$, inflation prevents U. collapse $\rightarrow$ black hole

=:-o a bad thing?

quantum gravity a prerequisite for inflation models?

moves away Guth's original idea, GUT physics?

★ ϵ, η also can quantify conditions for *ending* inflation

Amount of Inflation

during inflation scale factor grows exponentially
(in most models); in any case
quantify "amount" of inflation as
$N = \ln(a_{\rm fin}/a_{\rm init})$: number of "$e$-foldings"

What is needed?
to solve horizon, flatness, monopoles back to GUT scale:
$N \gtrsim N_{\rm min} \sim 60$ (PS6)

What is predicted?
Since $H = \dot{a}/a = d \ln a/dt = \dot{N}$, and $dt = d\phi/\dot{\phi}$, we have

$$N = \int_{t_{\rm init}}^{t_{\rm fin}} H\, dt = \int_{\phi_{\rm init}}^{\phi_{\rm fin}} \frac{H\, d\phi}{\dot{\phi}} \qquad (16)$$

slow roll: $\dot{\phi} \simeq -V'/3H$, so

$$N = \int_{\phi_{\rm fin}}^{\phi_{\rm init}} \frac{3H^2\, d\phi}{V'} = m_{\rm pl}^2 \int_{\phi_{\rm fin}}^{\phi_{\rm init}} \frac{V}{V'} d\phi \qquad (17)$$

typically expect $V'/V \sim 1/\phi$, which gives

$$N \sim \frac{\Delta\phi^2}{m_{\rm pl}^2} \qquad (18)$$

amount of inflation set by:

- nature of potential V
- change in ϕ

note also that need $N \gg 1$ and thus
typically expect $\phi_{\rm init} \gtrsim m_{\rm pl}$
...but already required by slow roll

Q: what determines inflation end physically? mathematically?

A Graceful Exit from Inflation

inflaton continues until acceleration stops ($w_\phi > 0$)
$\rightarrow$ potential energy no longer dominates cosmic ρ
all matter and radiation inflated away, so "rescue" comes from kinetic energy $\dot{\phi}^2/2$ (by itself, has $w = +1$!)

in terms of potential, exit when slow roll stops
quantified by slow-roll parameters
i.e., ϕ evolves until $\epsilon(\phi) \sim 1$

inflaton requirements:

- to achieve slow roll $\rightarrow$ need flat V far from minimum
- to end slow roll $\rightarrow$ need non-flat
  $V' \gtrsim V/m_{\rm pl}$ approaching minimum

Q: and then...? What's the Universe like? What happens next?

Reheating: Back to the Hot Big Bang

After $e^{60} \sim 10^{26}$ expansion
radiation, matter particles diluted to negligibility as a^{-3}
temperature drop $T \sim 1/a \to 0$: "supercooling"

But since $V(\phi) \sim const$ during inflation
inflaton energy density still large afterwards
must convert to hot, radiation-dominated early U: **reheating**

Details complicated, model-dependent; basic idea:
⋆ ϕ evolves in non-inflationary way
⋆ quantum effects drive energy conversion

Inflation and the Rest of Cosmology

Reheating Temperature

⋆ All of ‘usual” hot big bang begins *after* reheat

⋆ Must reheat enough for U to undergo
any and all known hot big phases
e.g., have to *at least* heat up to have nucleosynthesis
i.e., successful nuke requires $T_{\text{reheat}} > 1$ MeV
earlier phases (if any) demand hotter reheat

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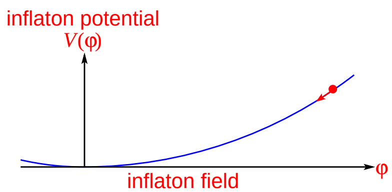

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7 (bonus) what about acceleration and dark energy today?

Q: what can we say about how inflation fits in the sequence *of cosmic events, e.g. monopole production, baryon genesis, BBN, CMB?*

Cosmic Choreography: The Inflationary Tango

Inflation must occur such that it
solves various cosmological problems, then
allows for the universe of today, which *must*

- ▷ contain the known particles, e.g., a net baryon number
- ▷ pass thru a radiation-dominated phase (BBN)
  and a matter-dominated phase (CMB, structure formation)

$\Rightarrow$ this forces an ordering of events

Cosmic Choreography: Required *time-ordering*

1. monopole production (if any)
2. inflation
3. baryogenesis (origin of $\eta \neq 0$)
4. radiation $\rightarrow$ matter $\rightarrow$ dark energy eras

Electroweak woes: hard to arrange baryogenesis afterwards!

Director's Cut Extras

Reheating I: Inflaton Oscillations

near minimum $V \simeq \frac{1}{2}V''\phi^2 \equiv \frac{1}{2}m_\phi^2\phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + V' \approx \ddot{\phi} + m_\phi^2\phi = 0 \quad (19)$$

simple harmonic oscillator!

▷ classically field oscillates around zero rapidly
and coherently: within particle horizon, same oscillation phase
so $\left\langle \dot{\phi}^2/2 \right\rangle = \langle V \rangle = \left\langle m_\phi^2\phi^2/2 \right\rangle$

- which means $\left\langle P_\phi \right\rangle = 0$ *Q: why?*
- which means $w_\phi = 0$, and so

$\left\langle \rho_\phi \right\rangle \sim a^{-3(1+w_\phi)} = a^{-3}$ like NR matter!
$\Rightarrow$ so ρ_ϕ drops $\to$ oscillation amplitude decays

Reheating II: Downfall of the Inflaton

▷ quantum mechanically field excitations → quanta
inflaton particles (mass m_ϕ) created

But the inflaton must be unstable *Q: why?*
→ decays to particles with Standard Model interactions
- if ϕ only decays to fermions
  does so slowly, products made thermally
- if ϕ can decay to bosons, resonances likely
  rapid decay far from equilibrium

In either case: decay products interact, exchange energy
thermalize: $\rho_\phi \rightarrow \rho_{\rm rad} \sim T^4$

$T_{\rm reheat} \sim \rho_{\phi,{\rm fin}}^{1/4}$

Q: what is rock-bottom minimum $T_{\rm reheat}$?