Astro 507
Lecture 33
April 17, 2020

Announcements:

- **Preflight 6 was due today**
- **Problem Set 6 due next Friday April 24**
  after this: final Problem Set due Finals Week
  recall: lowest PF and PS dropped

Last time: quantum effects in inflation
Today: inflation tests
and begin struture formation

Inflation Spectrum
Statistical Properties

- ⋆ Recall: inflaton quantum modes ↔ harmonic oscillator
  dominated by vacuum ↔ ground state $\|\psi_{\text{sho}}(x)\|^2 \sim e^{-x^2/2\Delta x^2}$
  $\phi_k \leftrightarrow x$ fluctuations are statistically Gaussian
  i.e., perturbations of all sizes occur, but
  probability of finding perturbation of size $\delta(R)$
  on scale R is distributed as a Gaussian

- ⋆ inflaton perturbations → reheating
  → radiation, matter perturbations
  same levels in both: "adiabatic"

⋆⋆⋆⋆⋆ All of these are bona fide *pre*dictions of inflation
and are testable! *Q: how?*

Inflation Spectrum
Slightly Tilted Scale Invariance

recall: perturbation leaving horizon have very similar amplitude during inflation → nearly same for all lengthscales ↔ k
perturbation rms amplitude

$$\delta^2_{\mathrm{inf}}(k) \propto k^{\alpha} \tag{1}$$

with index $\alpha = -6\epsilon + 2\eta \ll 1$

$\star$ successful inflation $\Leftrightarrow$ slow roll $\Leftrightarrow$ $\epsilon, \eta \ll 1$ demands
perturbation spectrum nearly independent of scale
nearly "self-similar," without characteristic scale
"Peebles-Harrison-Zel'dovich" spectrum

$\star$ successful inflation must end $\rightarrow$ $\epsilon, \eta \neq 0$
demands small departures from scale-invariance
"tilted spectrum"

Inflation Creates Primordial Gravity Waves

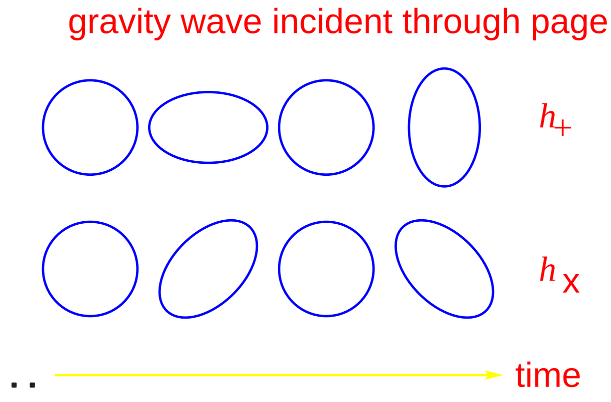
Inflaton field fluctuations are inhomogeneous perturbations to cosmic mass-energy density field

can excite **gravitational radiation**

when fluctuations have nonzero quadrupole, i.e., **tensor modes**

- **cosmic gravitational wave background**
- wavelengths span all scales up to Mpc
- wave amplitude directly related to density perturbations
- waves propagate unimpeded through Universe after inflation

effect on ring of test particles



Q: how to test?

Searching for Primordial Gravitational Waves

- waves drive quadrupole motion
  introduce **CMB polarization**
  we'll see: gravitational wave excite B modes–curl features

- In principle: direct detection possible via spacetime effects!
  but cosmo signal below astro events (BH, NS)
  not accessible to aLIGO/VIRGO, likely not LISA.
  `www: gravitational wave signal comparison`

Testing Inflation: Status to Date

test inflation by measuring density fluctuations
and their statistical properties
on various scales at various epochs

CMB at large angles (large scales, decoupling)

- nearly scale invariant! woo hoo! (COBE 93)
- Gaussian distribution (COBE, WMAP, Planck)
  `www: 3-yr WMAP` T `distribution`
- WMAP, Planck: evidence for tilt! favors large scales (“red”)!
  Planck (2013): $\alpha = -0.035 \pm 0.004$ nonzero at $\sim 9\sigma$!

These did not have to be true!
Not guaranteed to be due to inflation
but very encouraging nonetheless

Inflation Scorecard

As designed (postdictions) inflation solve:

- ✓ **horizon problem**
- ✓ **flatness problem**
- ✓ **smoothness problem**
- ✓ **monopole problem**

But *unexpected bonus*: structure
Thus far: observed cosmic density fields
have spectrum, statistics as predicted by inflation

- ✓ **nearly scale invariant**
- ✓ **gaussian statistics**
- ✓ **small tilt**

Frontier: CMB polarization probes of cosmic gravity waves
Stay tuned!

Intermission: Questions?

The Inhomogeneous Universe

Origin and Evolution of Cosmic Structure

The Large-Scale Structure of the Universe
Theoretical and Observational Landscape

On large scales, cosmo principle an excellent approximation
On small scales, fails miserably
Cosmology should explain both: now open our eyes to structure

Theory *Goals? tools? complications?*
Which scales in space, time "easy" to describe? which difficult?

Observations
Goals? observables? complications?
Which scales in space, time "easy" to measure? which difficult?

Arenas for theory–observation comparison
Which well-matched (i.e., clear results from both)?
Which poorly-matched (i.e., one or both ambiguous/difficult)?
What constitutes success? When are we done?

Large-Scale Structure: The Good, the Bad, and the Ugly

Structure Formation Theory

Goal: describe how small density fluctuation “seeds”
grow to form structure today

Tools: baryon-DM-radiation-DE particle & fluid dynamics
in expanding FLRW background
analytic–linearized perturb theory, idealized nonlinear models
numerical–full nonlinear evolution, feedback effects

Complications: nonlinear processes
(virialization, shocks, star feedback)

Degree of Difficulty:
large scales easiest–smoothest, linear perturb theory accurate
smallest scales hardest–very nonlinear

Structure Formation Observations

Goal: measure growth of structures over cosmic history

Tools: CMB anisotropy

surveys (optical, X-ray, IR, radio, γ-ray...): galaxies, quasars,
  QSO absorption systems, lensing

Complications: need for statistical completeness
  vs sensitivity, resolution

large scales easy in some ways: CMB very clean
  galaxy, quasar statistics best over largest volumes

...but difficult in others: sensitivity, resolution lowest
  few independent samples of structure at largest scales
  "cosmic variance" (e.g., see many 10 Mpc regions,
    only one at 4 Gpc)
  reshifting, absorption present challenges
  only a few epochs accessible

small scales easy in some ways: can probe locally
  sample many independent regions
  accessible at different epochs

...but difficult in others: hard to measure at large z

Comparing Theory and Observation

Strong Tests

well-matched at large scales:
linear theory accurate, observations (esp CMB) clean

Mismatches

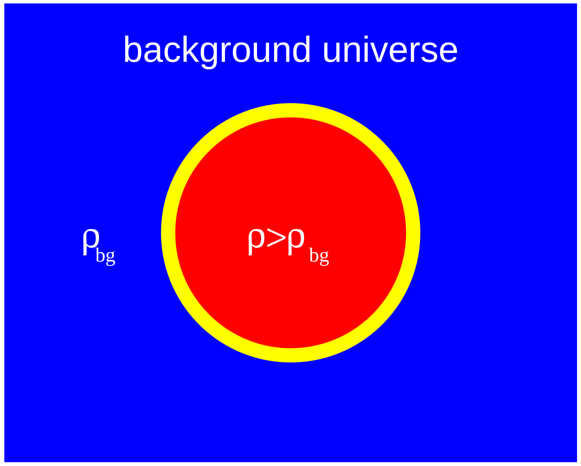
Theory naturally describes density evolution
dominated by dark matter–invisible!
Observations naturally look at light
easiest to look at most nonlinear, baryonic systems
Problem: *mass* vs *light* disconnect
“bias” – rarest=largest structures easiest to see
and baryons collisional, dissipative
→ more spatially concentrated that DM (think halos!)
Also: most light from stars–but theory of star form
incomplete and uncertain
⇒ *this is the frontier!*

Building Intuition: Spherical Collapse

consider idealized initial conditions

"top hat" Universe

- spherical, uniform density ρ
- embedded in flat, matter-dom universe with "background" density $\rho_{\rm bg}$ ("compensated" by surrounding underdense shell)



spherical collapse model a cosmological workhorse

a nonlinear problem with analytic solution!

Given: initial density contrast $\delta_i \ll 1$ at some t_i

Want to calculate: density contrast $\delta(t)$

lucky break–Newton's "iron sphere"/Gauss' law/Birkhoff's: in spherical matter distribution, interior ignorant of exterior

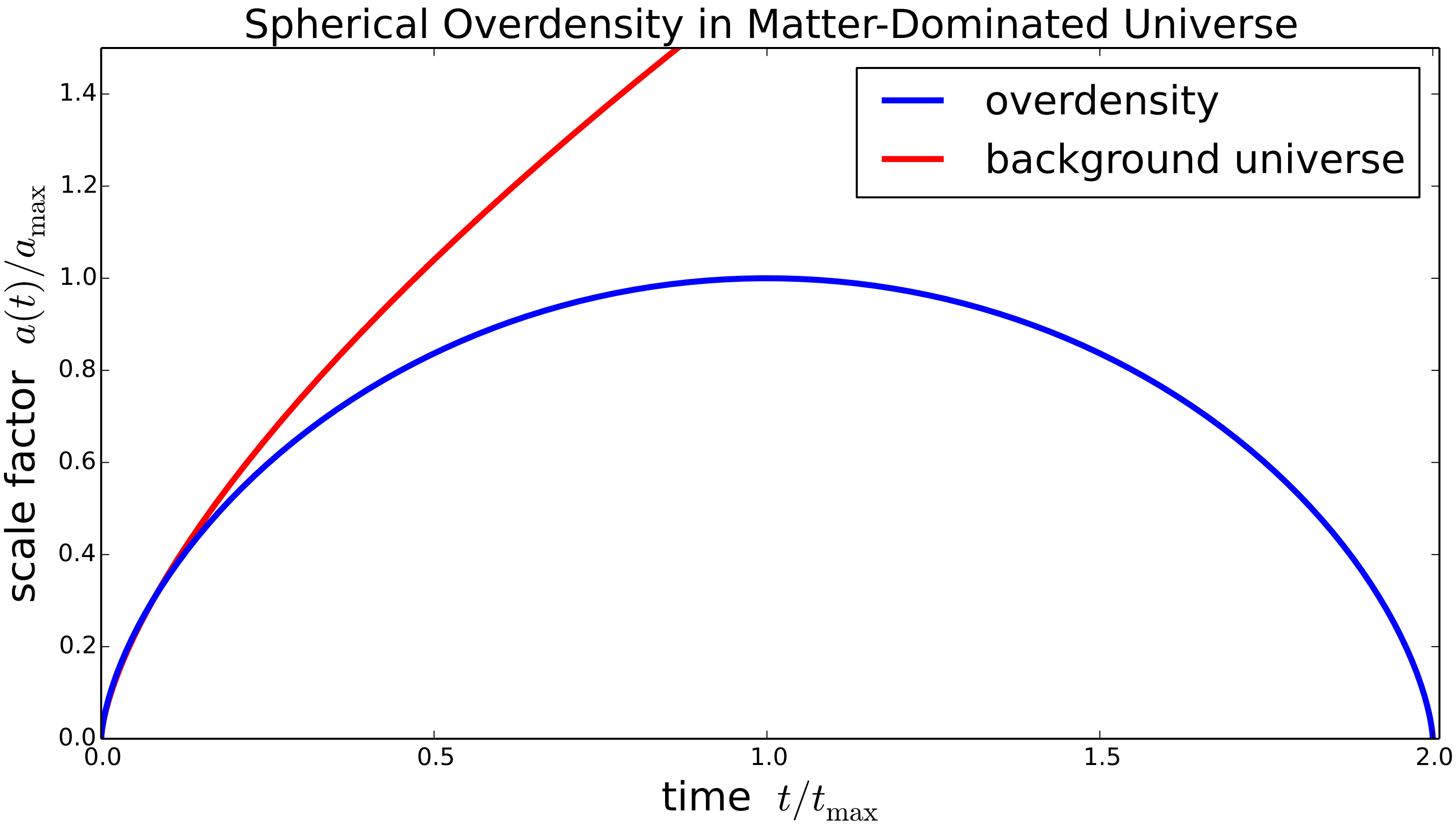
$\Rightarrow$ overdense region evolves exactly as closed universe!

PS6: solution is parametric (cycloid)

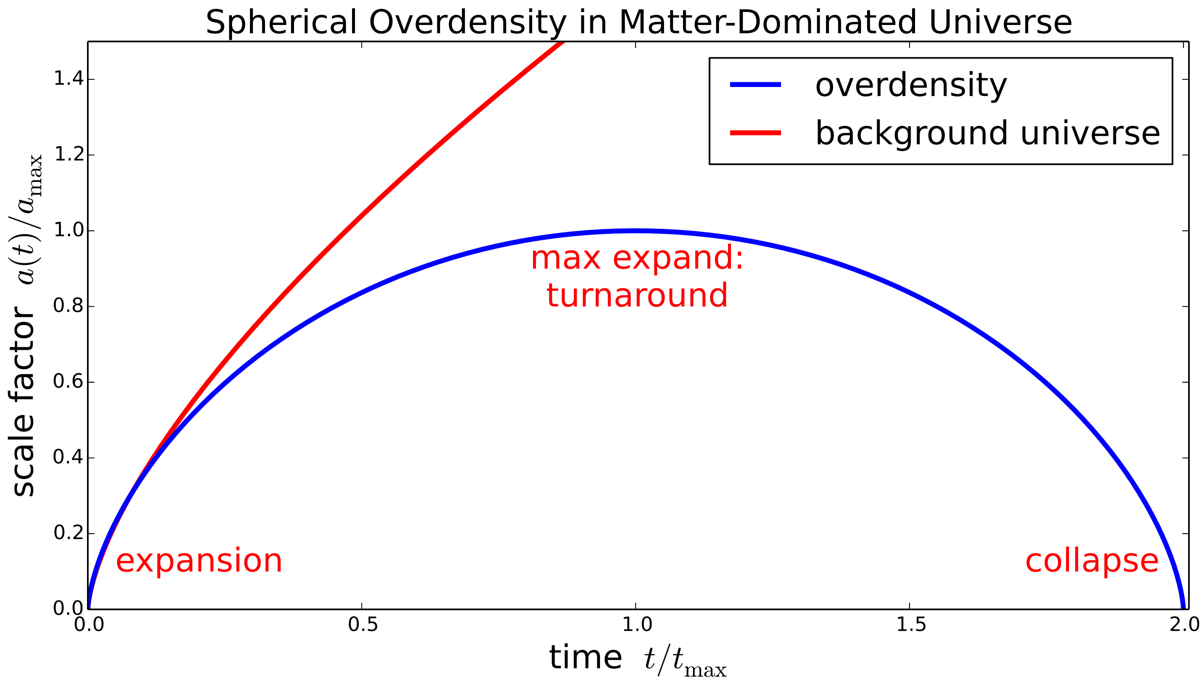
$$\begin{aligned} a(\theta) &= \frac{a_{\mathrm{max}}}{2}(1-\cos\theta) && (2) \\ t(\theta) &= \frac{t_{\mathrm{max}}}{\pi}(\theta-\sin\theta) && (3) \\ & && (4) \end{aligned}$$

evolution parameter: "development angle" θ

Q: a, t *for* $\theta = 0$*?* $\theta = \pi$*?* $\theta = 2\pi$*? Q: so what will this look like?*



Q: describe overdensity evolution qualitatively?



- initially expand with Universe
- but extra gravity in overdensity slows expansion
- reach max expansion at $t_{\rm max}$, then begin collapse
  "turnaround" epoch
- formally, collapse (to a point!) at $t_{\rm coll} = 2t_{\rm max}$

Q: what really happens when $t \gtrsim t_{\rm coll}$?

Spherical Collapse: Fate in Real Universe

Formal spherical collpase final state: *collapse to a point!*
"subuniverse" goes to big crunch!

- in reality: after turnaround, infalling matter virializes
  marks birth of halo as collapsed object
- Note: Brooklyn is not expanding! Nor is SS, MW, LG
  Q: what is criterion not *to expand?*

Beyond the formal solution:

- after virialized, halo still overdense
  → neighboring shells fall in
  → mass continues to grow by accretion!
- in real life: mergers too

Director’s Cut Extras

Gravity Waves: Tensor Perturbations

⋆ so far: only looked at density (scalar) perturbations
but also tensor perturbations → gravity waves!

what's really going on: *cosmic metric* is perturbed
spatial part (in a particular coordinate system = gauge):

- unperturbed = FLRW

$$d\ell^2|_{\mathrm{FLRW}} = a(t)^2 \ (dx^2 + dy^2 + dz^2) = a(t)^2 \ \delta_{ij} \ dx_i \ dx_j \tag{5}$$

with perturbations

$$d\ell^2|_{\mathrm{pert}} = a(t)^2 \ e^{2\zeta} \ \gamma_{ij} \ dx_i \ dx_j \tag{6}$$

with *curvature perturbation* the *scalar* function $\zeta(\vec{x}, t)$
Q: what it its physical effect?

perturbed metric

$$d\ell^2|_{\mathsf{pert}} = a(t)^2\ e^{2\zeta}\ \gamma_{ij}\ dx_i\ dx_j \tag{7}$$

curvature perturbation scalar function $\zeta(\vec{x},t)$ changes local volume

$\rightarrow$ locally: isotropic stretching

tensor perturbation is, to lowest order

$$\gamma_{ij} \approx \begin{pmatrix} 1+h_+ & h_\times & 0 \\ -h_\times & 1-h_+ & 0 \\ 0 & 0 & 1 \end{pmatrix} = \delta_{ij} + \begin{pmatrix} h_+ & h_\times & 0 \\ -h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

with *two independent modes* of amplitude $h_+, h_\times$

Q: physical effect of these modes?

tensor perturbation is, to lowest order

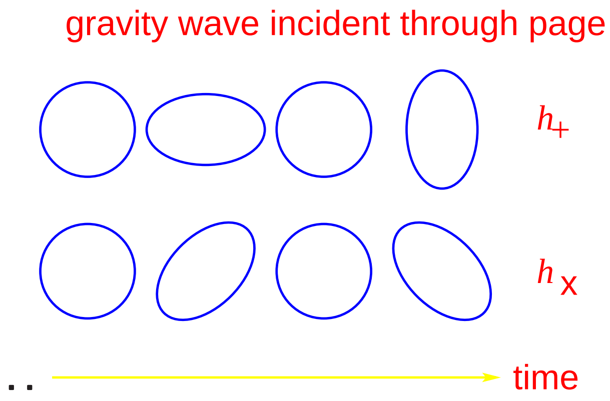
$$\gamma_{ij} \approx \delta_{ij} + \begin{pmatrix} h_+ & h_\times & 0 \\ -h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (9)$$

looks like rotation: roughly speaking preserves volume but changes angles

moreover: h satisfies massless wave equation!

$h \Leftrightarrow$ **gravitational radiation**

effect on a ring of test particles:



Metric Fluctuations

tensor perturbations directly are metric perturbation
what about the inflaton perturbations?

curvature perturbation in an invariant (coordinate independent):

$$\zeta = \Phi + H\delta t = \Phi + H\frac{\delta\phi}{\dot\phi} \tag{10}$$

$\Phi(\vec{x},t)$ is local *gravitational potential perturbation*

inflation fluctuations ϕ also are metric perturbations
but amplitude differs from gravity wave amplitude
by factor $H/\dot\phi$
and thus scalar perturbation variance differs by factor

$$r = \frac{\Delta_h^2}{\Delta_\Phi^2} \sim \left(\frac{\dot\phi}{H}\right)^2 \sim \epsilon \tag{11}$$

Inflationary Tensor Perturbations

variance as a function of scale (wavenumber)

$$\Delta_h^2(k) \sim \left(\frac{V}{m_{\mathrm{pl}}^4}\right)_{aH=k} \tag{12}$$

- evaluated at “horizon crossing” $aH = k$
- **directly probes inflation potential $V(\phi)$!**
- compare to density (“scalar”) perturbations:

  tensor-to-scalar ratio

$$r = \frac{\Delta_h^2}{\Delta_\Phi^2} = 16\epsilon \tag{13}$$

- for $\epsilon \ll 1$, expect $r \ll 1$: scalar dominates