Astro 507

Lecture 34

April 20, 2020

Announcements:

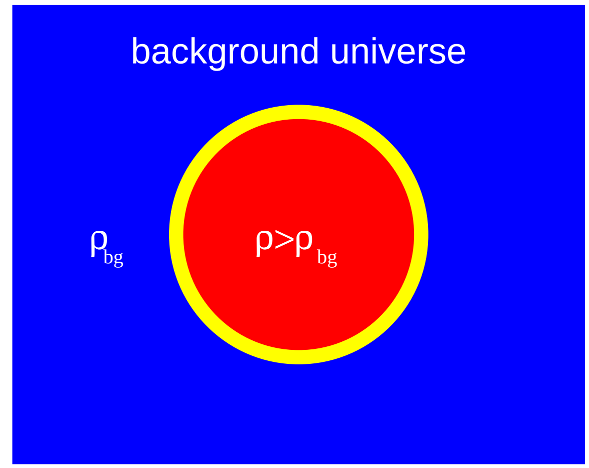
- **Problem Set 6 due next Friday April 24**
  after this: final Problem Set due Finals Week
  recall: lowest PF and PS dropped
- Preflight 6a comments posted
  feel free to talk to me if you have questions
  or find the scope hard to manage

Last time: Welcome to the inhomogeneous universe
presents a wealth of new cosmology probes
at the cost of more complexity observationally and theoretically

Building Intuition: Spherical Collapse

consider idealized initial conditions
"top hat" Universe

- spherical, uniform density ρ
- embedded in flat, matter-dom universe with "background" density $\rho_{\rm bg}$ ("compensated" by surrounding underdense shell)



spherical collapse model a cosmological workhorse
a nonlinear problem with analytic solution!

Given: initial density contrast $\delta_i \ll 1$ at some t_i
Want to calculate: density contrast $\delta(t)$
lucky break–Newton's "iron sphere"/Gauss' law/Birkhoff's:
in spherical matter distribution, interior ignorant of exterior
$\Rightarrow$ overdense region evolves exactly as closed universe!

PS6: solution is parametric (cycloid)

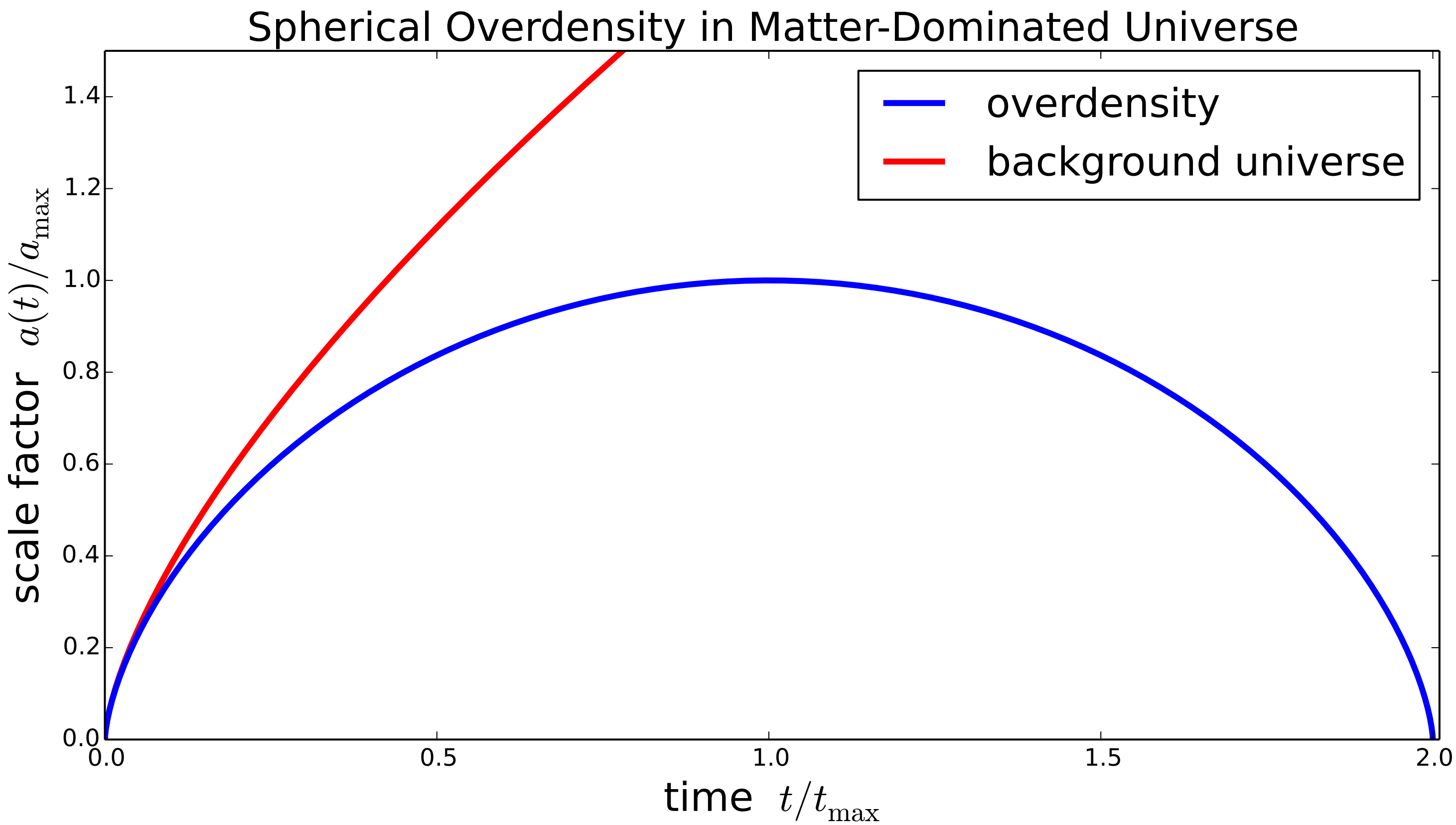
$$a(\theta) = \frac{a_{\rm max}}{2}(1 - \cos\theta) \quad (1)$$

$$t(\theta) = \frac{t_{\rm max}}{\pi}(\theta - \sin\theta) \quad (2)$$

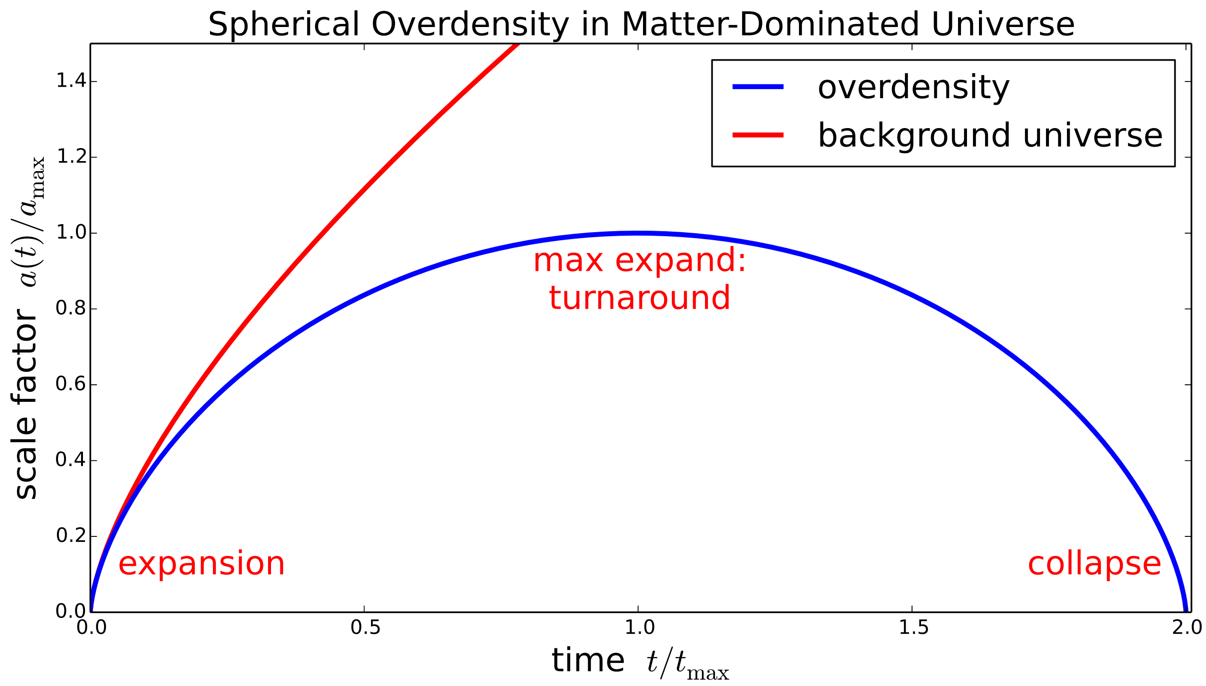
$$(3)$$

evolution parameter: "development angle" θ

Q: a, t for $\theta = 0$? $\theta = \pi$? $\theta = 2\pi$? Q: so what will this look like?



Q: describe overdensity evolution qualitatively?



- initially expand with Universe
- but extra gravity in overdensity slows expansion
- reach max expansion at $t_{\rm max}$, then begin collapse
  "turnaround" epoch
- formally, collapse (to a point!) at $t_{\rm coll} = 2t_{\rm max}$

Q: what really happens when $t \gtrsim t_{\rm coll}$?

Spherical Collapse: Fate in Real Universe

Formal spherical collapse final state: *collapse to a point!*
"subuniverse" goes to big crunch!

- in reality: after turnaround, infalling matter virializes
  marks birth of halo as collapsed object
- Note: Brooklyn is not expanding! Nor is SS, MW, LG
  Q: what is criterion not *to expand?*

Beyond the formal solution:

- after virialized, halo still overdense
  $\rightarrow$ neighboring shells fall in
  $\rightarrow$ mass continues to grow by accretion!
- in real life: mergers too

Intermission: Questions?

Quantifying Large-Scale Structure

Observed galaxy distribution random

▷ location, form of individually galaxies unpredictable
  but clearly correlations, characteristic scales

▷ reflects randomness of initial conditions

▷ demands a fundamentally statistical treatment

Statistical description of cosmic density fields

consider, e.g., mass density $\rho(t, \vec{x})$

not only *random*, but also *continuous*

yet most observations are of *discrete* objects
  galaxies, clusters, etc.

`www: slices of the Universe`

∞

Q: how to address this?

Method I: Fluctuations of Counts in Cells

fix a lengthscale $\lambda \to$ volume $V = \lambda^3$
divide patch of U. into cells of this size

then can define avg density $\langle \rho_i \rangle$ in each box i
  or more observationally: galaxy count N_i in box
  then look at statistical properties of N_i distribution

assume: different boxes $\langle \rho_i \rangle$, $\langle \rho_k \rangle$ initially indep
quickly independence lost *Q: why?*

but want a characterization in which different elements
  ("realizations") are independent
Q: how to do this?

Problem: neighboring cells affect each other
e.g., overdensities drain underdensities next door
$\rightarrow$ evolution immediately couples cells

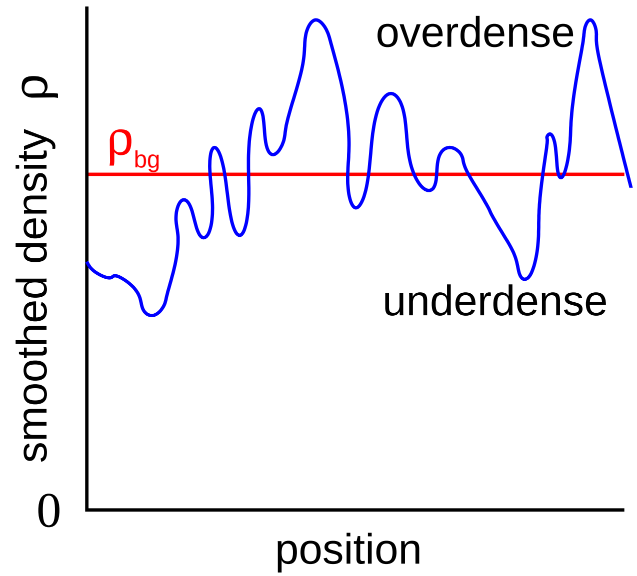
this is useful, and is done

but worthwhile to find more approaches:
all have some limitations

Q: suggestions?

Quantifying Structure: Smooth Density Field

- fix a lengthscale λ
  $\rightarrow$ volume $V \sim \lambda^3$
- *smooth* or *coarse-grain*
  the density over this scale



- study **density fluctuations** around cosmic mean
  i.e., departures from "background" density $\rho_{\rm bg}(t)$
- repeat for different scales L

Q: How to quantify fluctuations?

Quantifying Density Fluctuations

Given $\rho(t, \vec{x})$, define

mean (average) density $\langle \rho \rangle = \langle \rho(t, \vec{x}) \rangle = \rho_{\mathrm{FRW}}(t)$: "background"
(suppress t hereafter)

density fluctuation $\delta\rho(\vec{x}) = \rho(\vec{x}) - \langle \rho \rangle$

density contrast

$$\delta(\vec{x}) = \frac{\delta\rho}{\rho} = \frac{\rho(\vec{x}) - \langle \rho \rangle}{\langle \rho \rangle} \tag{4}$$

where $\delta \neq \delta_{\mathrm{Dirac}}$!

Q: possible range of δ values?

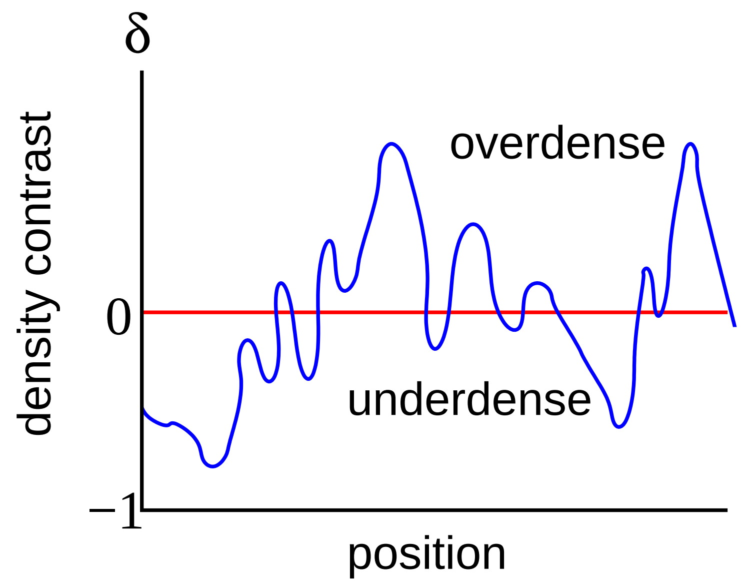
Q: what is $\langle \delta \rangle$?

Q: how does cosmic expansion affect δ?

key measure of cosmic structure:
density contrast

$$\delta(\vec{x}) \;=\; \frac{\delta\rho}{\rho} \equiv \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle} \in (-1,\infty)$$

where average is over large volume V



Q: what is the order-of-magnitude of the density contrast in this room? of the Galactic ISM?

by definition: $\langle\delta\rangle = \frac{1}{V}\int d^3x\,\delta(\vec{x}) = 0$

would like to study structures on different cosmic lengthscales λ
Q: how to do this using density contrast?

Spherical Collapse Revisited

in spherical collapse model we can calculate overdensity:
since $\rho \propto 1/a^3$

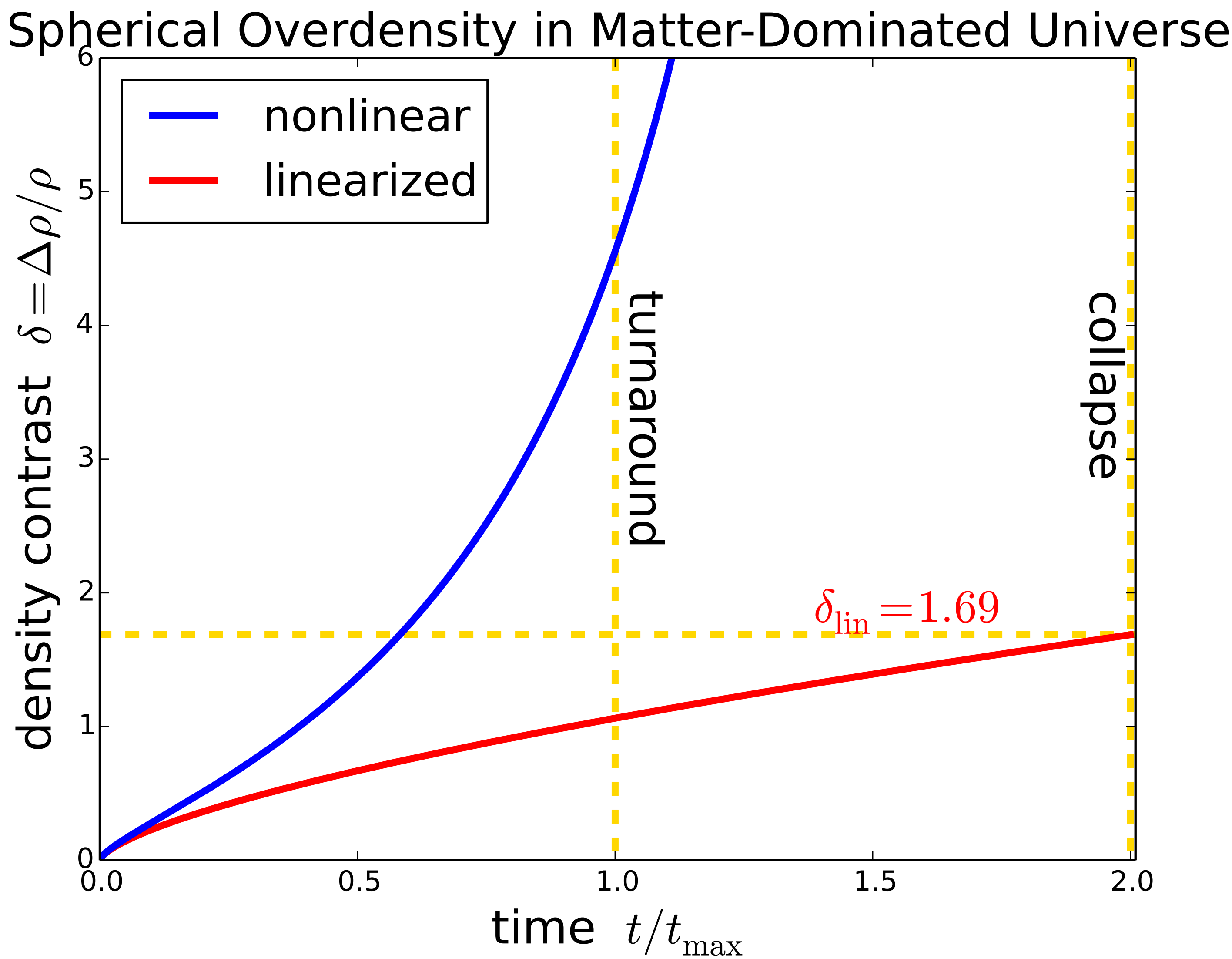
$$\delta(t) = \frac{\rho(t)}{\rho_{\rm bg}(t)} - 1 = \left(\frac{a_{\rm bg}}{a}\right)^3 - 1 \tag{5}$$

with $a_{\rm bg} \propto t^{2/3}$ the matter-dom background
$\rightarrow$ exact nonlinear solution (pre-virial)

For small t, to first order $a(t) \sim t^{2/3} = a_{\rm bg}(t)$:
recover background result; $\delta(t) = 0$
PS6–to second order: $a(t) = a_{\rm bg}(t)[1 - (12\pi t/t_{\rm coll})^{2/3}/20]$

$$\delta(t) \approx \frac{3}{20}\left(\frac{12\pi t}{t_{\rm coll}}\right)^{2/3} = \delta_{\rm lin}(t) \tag{6}$$

"linearized" density contrast: $\delta_{\rm lin}(t) \propto t^{2/3} \propto a_{\rm bg}$


Spherical Overdensity in Matter-Dominated Universe
nonlinear
linearized
turnaround
collapse
$\delta_{\rm lin} = 1.69$
density contrast $\delta = \Delta\rho/\rho$
time $t/t_{\rm max}$
0
1
2
3
4
5
6
0.0
0.5
1.0
1.5
2.0

Very useful result:

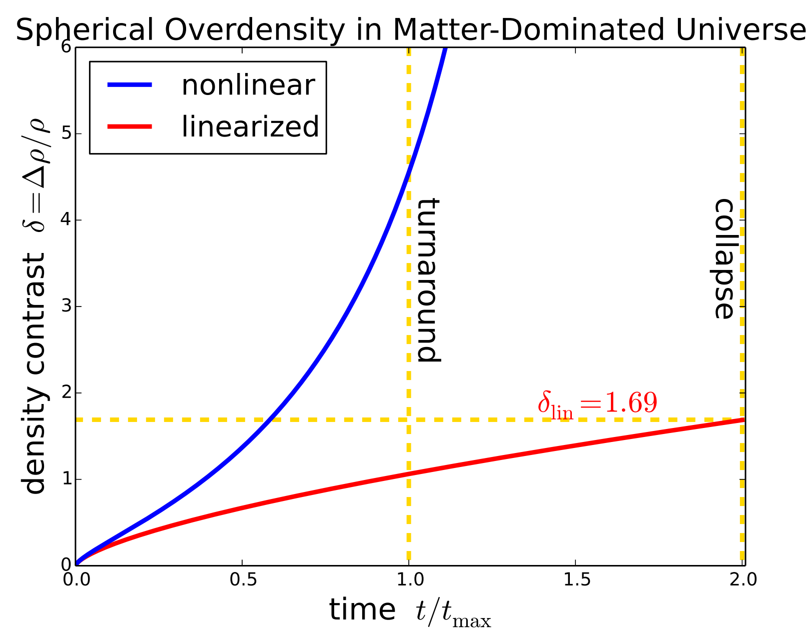
$$\delta_{\rm nonlin}(t) = \left(\frac{a_{\rm bg}}{a_{\rm nonlin}}\right)^3 - 1 \quad (7)$$

$$\delta_{\rm lin}(t) \approx \frac{3}{20}\left(\frac{12\pi t}{t_{\rm coll}}\right)^{2/3} \quad (8)$$

connects full nonlinear result with linear counterpart
$\rightarrow$ maps between the two

- at turnaround
  $\delta_{\rm nonlin} = (6\pi)^2/4^3 - 1 = 4.6$
  but $\delta_{\rm lin} = 1.06$
- including virialization (PS6):
  $\delta_{\rm nonlin} \approx 180$, but $\delta_{\rm lin} = 1.69$
  defines a critical linear overdensity

Q: why useful?

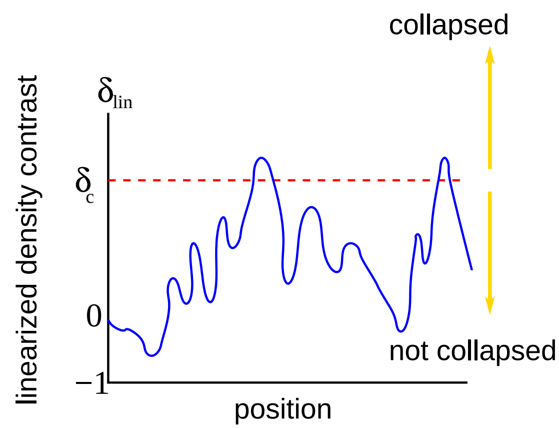


Strategy: *given initial linear density field* δ_i

- evolve perturbations with linear growth $\delta_{\text{lin}}(t)$
- identify linearly extrapolated perturbations with $\delta_{\text{lin}}(t) > 1.69$

  $\Rightarrow$ *these will be collapsed objects by time t!*

lesson: in *linearized* $\delta_{\text{lin}}(t_0)$
a "cut" at $\delta_c = 1.69$
divides virialized vs nonvirialized



also: in a *nonlinear field*, can use $\delta_{\text{vir}} \sim 180$
as working *definition* of collapsed structure
good for comparing theory, observation *Q: procedure?*

Nonlinear Evolution: Lessons from Spherical Collapse

Qualitatively

▷ overdensity evolves as closed "subuniverse"

▷ starts expanding, but slower than cosmic background
pulls away from Hubble flow: reach max expansion, then turnaround

▷ virialize $\rightarrow$ form bound object

▷ no further expansion, except due to accretion, merging

Quantitatively

▷ can compute both $\delta_{\rm lin}(t)$ and exact $\delta(t)$
gives mapping from *easy* to (more) *correct*

▷ collapse/virialization when $\boxed{\delta_{\rm lin} = 1.69 \text{ and } \delta = 18\pi^2 \simeq 180}$
recipe for forecasting structures in initial field $\delta_{\rm init} \ll 1$
recipe for defining halos: region surrounding density peak
and having overdensity $\delta\rho/\rho \sim 180$

★ Given these, can devise analytical tools to describe distribution of structures