Astro 507

Lecture 38

April 29, 2020

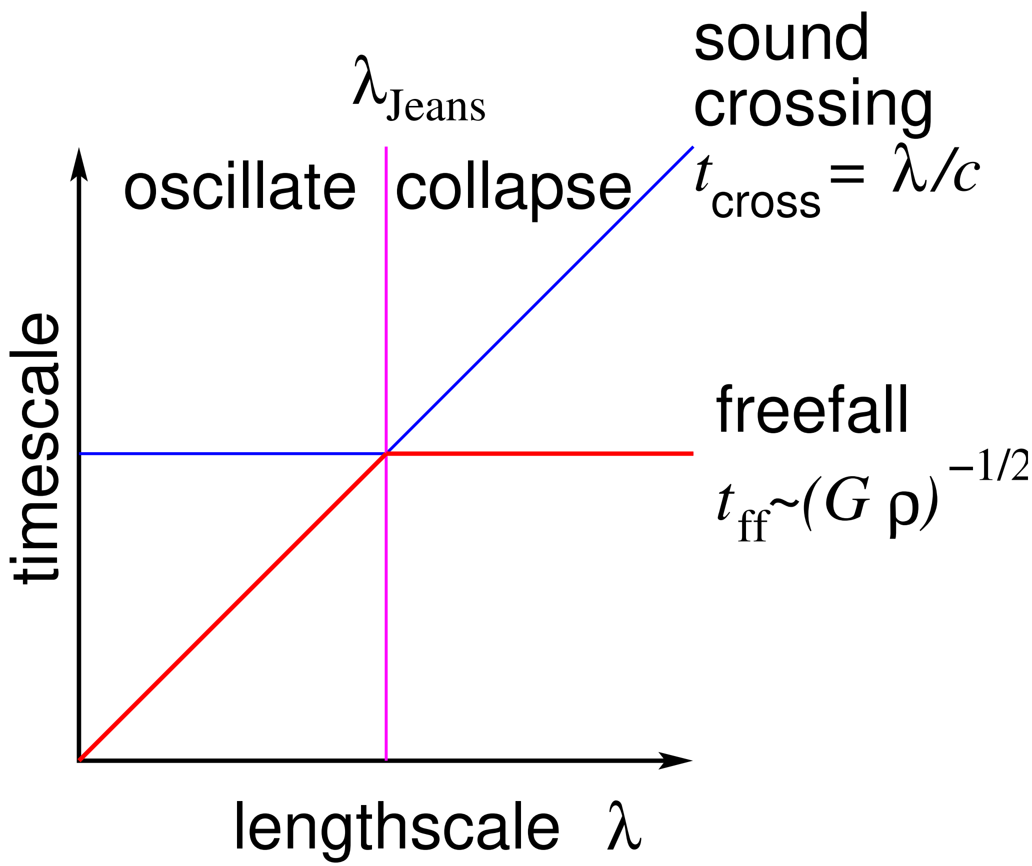
Announcements:

- **Preflight 6b due this Friday May 1**
  draft your Wikipedia upgrade, post for comments
  have fun, ask if you need advice/help

Last time: Jeans linear analysis of gravitational instability
for perturbations of wavelength λ and wavenumber $k = 2\pi/\lambda$
warmup: artificially static universe
Q: perturbation behavior $\delta_k(t)$ *on small scales? on large scales?*
Q: what determines which behavior–i.e., key scale?

Linear Perturbations: Static Universe

- small scales: $\delta_k(t) \propto \cos(c_s k t)$ oscillation
- large scales: $\delta_k(t) \propto \exp(+\omega_{\rm ff} t)$ collapse
- small/large set by **Jeans length** $\lambda_{\rm J} \sim c_s \tau_{\rm ff}$



now for expanding universe:

Q: similarities?

Q: differences?

Linear Perturbations: Expanding Universe

- small scales: oscillation
- large scales: $\delta_k(t) \propto t^{2/3}$ collapse
- small/large set by **Jeans length** $\lambda_{\mathrm{J}} \sim c_s \tau_{\mathrm{ff}}$
- qualitatively similar fates, Jeans length plays same role
- but cosmic expansion (“Hubble drag”) opposes collapse so unstable modes grow as power law, not exponentially

Linear Growth Factor

each unstable Fourier mode grows with time as

$$\delta_k(t) \propto D(t) \sim t^{2/3} \sim a \sim \eta^2_{\text{conform}} \qquad (1)$$

growth independent of wavenumber k

- in k-space, all unstable modes grow by same factor and transform to real space, find
- on large scales (but still subhorizon)

$$\delta(t, \vec{x}_{\text{large}}) \simeq D(t)\delta(t_i, \vec{x}_{\text{large}}) \qquad (2)$$

⇒ entire density contrast pattern grows with same amplification:

⇒ **linear grow factor** $D(t)$ applies to whole field

Q: what would this look like for $\delta(x)$? `www: animation`

Applications to CMB: Naïve Inferences

before decoupling: pressure dominated by photons
$\rightarrow$ expect oscillations – and see them!
after decoupling: growing mode

CMB anisotropies are a snapshot
of perturbations at last scattering
can quantify level: $(\delta T/T)_{\rm ls} \sim 10^{-5}$ at $z_{\rm ls} \sim 1100$

But matter has $\rho \propto a^{-3} \propto T^3$, so $\delta\rho/\rho = 3\delta T/T$
$\rightarrow$ $\delta_{\rm obs}(z = 1100) \sim 3 \times 10^{-5}$ at last scattering
So today, expect fluctuations of size

$$\delta_0 = \frac{D_0}{D_{\rm ls}}\delta_{\rm ls} = \frac{a_0}{a_{\rm ls}}\delta_{\rm ls} = (1 + z_{\rm ls})\delta_{\rm ls} \sim 0.05 \ll 1 \tag{3}$$

Should still be very small–no nonlinear structures, such as us!
Q: obviously wrong–egregiously naïve! What's the flaw?
What's the fix?

Perturbation Growth: Dark Matter vs Baryons

dark matter: pressureless
$\rightarrow$ all k modes unstable if inside Hubble length
but: perturbations grow slooooowly during radiation era (see Extras)
$\rightarrow$ DM structures begin formation at matter-radiation equality
then $\delta_{\rm m}(t) = \delta_{\rm m,init}\ D(t)$ with $D(t) \propto a(t) \propto t^{2/3}$

baryons: until recomb, tightly coupled to photons
$\rightarrow$ feel huge photon pressure $P_\gamma \propto T^4$
$\rightarrow$ sound speed $c_s \sim c/\sqrt{3}$ huge!
so all sub-horizon modes stable! just oscillate
$\rightarrow$ relativistic pressure-mediated (i.e., acoustic) standing waves!

Cosmic Diversity: Evolution of Multiple Components

Thus far: *implicitly assumed a baryons-only universe*: not ours!

Cosmic "fluid" contains many different species
with different densities, interactions
baryons, photons, neutrinos, dark matter, dark energy

Each component i has its own equations of motion, e.g.:

$$\ddot{\delta}_i + 2H\dot{\delta}_i = -\frac{c_{s,i}^2 k^2}{a^2}\delta_i + 4\pi G\rho_0 \sum_j \Omega_j \delta_j \tag{4}$$

species interact via pressure, gravity: evolution eqs *coupled*

▷ gravity from dominant Ω drives the other components

▷ each species' (pressure) response depends on
microphysics of its interactions, encoded in sound speed $c_{s,i}$

Intermission: Questions?

CMB Anisotropies

CMB Anisotropies

Between matter-radiation equality and recombination:

- dark matter perturbations grow
  form deepening potential wells
- baryons, electrons tightly coupled to photons (plasma)
  undergo oscillations: *gravity vs pressure = acoustic*

Q: what is the largest scale which can oscillate?
Q: for each mode k, what sets oscillation frequency?
Q: at fixed t, which scales have oscillated the most? the least?
Q: how is this written on the CMB?

Pre-Recombination: Acoustic Oscillations

Baryons in DM-dominated background

$$\ddot{\delta}_b + 2\frac{\dot{a}}{a}\dot{\delta}_b \simeq 4\pi G\rho\delta_{dm} - \frac{k^2c_s^2}{a^2}\delta_b \sim \frac{\delta_{dm}}{t^2} - \frac{k^2c_s^2}{a^2}\delta_b \quad (5)$$

key comparison: mode scale $\lambda \sim k^{-1}$
vs **comoving sound horizon** $c_s t/a = d_{s,com}$

for *large scales* $kc_s t/a \ll 1$: *baryons follow DM*
for *small scales* $kc_s t/a \gg 1$: *baryons oscillate*, as

$$\delta_b \sim \frac{1}{\sqrt{ac_s k}} e^{i\int kc_s d\eta} \quad (6)$$

(PS 6) where $d\eta = dt/a$ is conformal time

Q: for fixed k, what is δ time behavior?
Q: at fixed t, what is δ pattern vs k?
Q: what sets largest λ that oscillates?

baryonic perturbations do not grow, but oscillate:

$$\delta_b \sim \frac{1}{\sqrt{a c_s k}} e^{i \int k c_s d\eta} \qquad (7)$$

to simplify, imagine constant c_s, $\delta_b \sim e^{i k c_s \eta}$

at fixed k, sinusoidal oscillations

phase counts number of cycles $N = k c_s \eta / 2\pi = c_s \eta / \lambda$

oscillation frequency: $\omega \sim k c_s \sim c_s / \lambda \propto 1/\lambda$

at fixed $t \rightarrow$ fixed η:

small λ and large $k \rightarrow$ rapid oscillations

largest oscillations at scale $\lambda \sim c_s \eta \sim c_s t / a$: *sound horizon*

Q: when do oscillations stop? observable signature?

Recombination: Snapshot Taken

At recombination, free e^- abundance drops

baryons quickly decouple from photons

huge drop in pressure $\rightarrow$ $c_s \rightarrow 0$

begin to collapse onto DM potentials

photons travel freely (typically) afterwards

fluctuation pattern at recomb is "frozen in"

δ vs scale records different # of cycles at recomb

$$P(k) = \|\delta_k\|^2 \sim \frac{\sin(2kc_s\eta_{\rm rec})}{2kc_s\eta_{\rm rec}} P_{\rm init}(k) \quad (8)$$

written onto temperature pattern ("say cheese!")

Recomb fast $\rightarrow$ CMB is image of last scattering surface

Q: on small scales, is an overdensity a hot spot or cold spot? why?

Spots Cold and Hot: Small Scales

Define temperature fluctuation $\Theta = \delta T/T$

On Small Scales: Adiabatic

standing waves lead to fluctuations in $\rho_b \sim T^3$, so

$$\Theta \equiv \frac{\delta T}{T} = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_b \tag{9}$$

$\Rightarrow$ extrema in density $\rightarrow$ extrema in $\Theta \propto \delta_\gamma$

$\star$ photon T contrast reflects T distribution at source

- but both high *and* low density give *large* $(\delta T/T)^2$!
  photon climb out of potential doesn't change $\delta T/T$ much
  $\rightarrow$ CMB hot spots are high density, cold are low

Q: what about on large scales?

Very Large Scales: Sachs-Wolfe

beyond horizon: no oscillations, main effects gravitational (GR):

- gravitational redshift: photon climbs out of potential $\delta\Phi < 0$
  redshift $\delta\lambda/\lambda = \Phi_0 - \Phi_{\rm ls} = -\delta\Phi$
  and since $T \sim 1/\lambda$, $(\delta T/T)_{\rm redshift} = \delta\Phi$: photons cooled!
- time dilation: takes longer to climb out of overdensity
  looking at younger, hotter universe
  $\delta t/t = \delta\Phi$, and since $a \sim t^{2/3}$ and $T \sim 1/a$
  then $T \sim t^{-2/3}$, and $(\delta T/T)_{\rm dilation} = -2/3\ \delta\Phi$

net effect: Sachs - Wolfe

$$\left(\frac{\delta T}{T}\right)_{SW} = \left(\frac{\delta T}{T}\right)_{\rm redshift} + \left(\frac{\delta T}{T}\right)_{\rm dilation} = \frac{1}{3}\delta\Phi \qquad (10)$$

★ overdensities are cold spots, underdensities hot

Note: this regime is what tests inflation

Q: what predicted?

Inflation and Sachs-Wolfe

Inflation: quantum fluctuations $\rightarrow$ density fluctuations

- adiabatic (all species)
- Gaussian
- scale invariant–what does this mean?

Extras: inflation scale invariance for wavenumber k
sets "power spectrum" – mean-square flucuation at k

$$\left\langle |\delta_k|^2 \right\rangle = P_{\text{scale-inv}}(k) = A k^{n_{\text{scale-inv}}} \tag{11}$$

with A a constant (sets flucutaions amplitudes)
and $n_{\text{scale-inv}} = 1$

Predictions:

- fluctuations occur on all scales
- largest amplitudes for big k $\rightarrow$ small scales
- $\delta_k \rightarrow 0$ for $k \rightarrow 0$, as we must find *Q: why?*

Angular vs Linear Scales

So far: decomposed fluctuations in (3-D) $\vec{k}$-space
but observe on sky: 2-D angular distribution

Transformation: projection of plane waves
at fixed k: see intersection of wave with last scattering shell
`www: Wayne Hu animation`

appears on a range of angular scales
but typical angular size is $\theta \sim \lambda/d_{\text{rec,com}} \sim (k d_{\text{rec,com}})^{-1}$

large angles $\rightarrow$ large λ (check!)
for large angular scales $\theta > \theta_{\text{hor,diam}} \sim 1^\circ$, superhorizon
perturbations not affected by oscillation
for small angular scales, see standing waves

- peaks at extrema, harmonics of sound horizon
  k are in ratios 1:2:3:...

The CMB Observed

- Observe 2-D sky distribution of $\frac{\Delta T}{T}(\hat{n}) \equiv \Theta(\hat{n})$ in direction $\hat{n}$
- Decompose into spherical harmonics

$$\Theta(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \tag{12}$$

with Y_{lm} spherical harmonics *Q: why not* $\ell = 0, 1$*?*
Q: angular size vs ℓ*?* λ *vs* ℓ*?*

Form angular correlation function *Q: what is this physically?*

$$\langle \Theta(\hat{n}_1)\Theta(\hat{n}_2) \rangle = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) \left\langle |a_{\ell m}|^2 \right\rangle P_\ell(\hat{n}_1 \cdot \hat{n}_1) \tag{13}$$

$$= \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) C_\ell P_\ell(\cos\vartheta) \tag{14}$$

where $\cos\vartheta = \hat{n}_1 \cdot \hat{n}_1$
Q: averaged over the m *azimuthal modes–why?*

all interesting anisotropy information encoded in

$$C_\ell = \left\langle |a_{\ell m}|^2 \right\rangle \tag{15}$$

isotropy $\rightarrow$ azimuthal dependence averages to zero

Note: analog of Δ^2 (variance per log scale) is
$\mathcal{T}^2(\ell) = \ell(\ell+1)C_\ell$: usually what is plotted

Since $P_\ell(\cos\theta) \sim (\cos\theta)^\ell \sim \cos(\ell\theta)$
at fixed ℓ, angular size $\theta \sim 2\pi/\ell = 180^\circ/\ell$
e.g., $\ell = 2$ quadrupole $\rightarrow \theta \sim 90^\circ$
and horizon size $\theta \sim 1^\circ$ is at $\ell \sim 200$

and since $\theta \sim \lambda/d_{\rm rec} \sim 1/dk$:
multipoles scale as $\ell \sim 1/\theta \sim k \sim 1/\lambda$
low $\ell \rightarrow$ big angular, physical scales $\rightarrow$ small k

CMB Anisotropy Observations: Strategy

- achieve high sensitivity, remove systematics
  make a "difference experiment"
  i.e., measure δT directly, don't subtract
- observe as much of the sky as possible (or as needed!)
  balloons/ground: limited coverage
  satellites (COBE, WMAP, Planck): all-sky
- remove Galactic contamination: "mask" plane
- recover Θ for observed region

- decompose into spherical harmonics $Y_{\ell m}$
- construct power spectrum $\ell(\ell+1)C_\ell$
- report results
- collect thousands of citations, prominent Prizes

CMB Temperature Anisotropies: Results

COBE (1993)

- first detection of $\delta T/T \neq 0$
- receiver horn angular opening $\sim 8^\circ$
  $\rightarrow$ only sensitive to large angular scales
  i.e., superhorizon size
- found $(\delta T/T)_{\rm rms} \sim 10^{-5}$
- power $\ell(\ell+1)C_\ell$ flat $\rightarrow$ implies $P(k) \sim k$!
  $n = 1$ spectrum: scale invariant!

Interregnum (late 90's, early 00's)

- ground-based, balloons confirmed anisotropy
- acoustic peaks discovered
  strong indication of first peak

WMAP (2003-)

- first all-sky survey of small angular scales
- $n = 1$ confirmed, indication of small tilt $n - 1 \neq 0$?
  consistent with inflation! and non-trivially so!
- acoustic peaks mapped: good measurement of 1st, 2nd
  detection of third
- first peak: $\ell \sim 200$ horizon at recomb!
- power dropoff seen at large ℓ
  $\rightarrow$ nonzero thickness of last scattering
  due to photon diffusion, non-instantaneous decoupling

Director's Cut Extras

Matter Instability in the Radiation Era

(dark) matter perturbation δ_m during radiation domination

- pick subhorizon scale: growth possible
- focus on $k < k_J$: Jeans unstable (can ignore pressure) and high-k modes just oscillate anyway
- treat radiation perturbations as *smooth*: $\delta_{\rm rad} \approx 0$ $P_r = \rho_r/3$: huge, fast $c_s \sim c$ any perturbations will be oscillatory anyway
- dark matter: weak interactions $\rightarrow$ pressureless $\rightarrow$ $c_s = 0$!

Evolution simple – to rough approximation, for these k:

$$\ddot{\delta}_m + 2\frac{\dot{a}}{a}\dot{\delta}_m \stackrel{\rm rad-dom}{=} \ddot{\delta}_m + \frac{1}{t}\dot{\delta}_m \approx 0 \qquad (16)$$

Simple solutions: growing mode plus decaying mode

$$\delta_m(t) = D(t)\delta_m(t_i) = \left(D_1 \log t + \frac{D_2}{t}\right)\delta_m(t_i) \qquad (17)$$

Q: implications? what about baryons?

Found $D(t) \sim D_1 \log t$: "growing" mode hardly grows!

$\star$ dark matter perturbations *frozen* during rad dom
dark matter growth quenched by
$\rightarrow$ non-growth of radiation perturbations
$\rightarrow$ extra expansion due to radiation

$\star$ *dark matter perturbation growth stalled*
until end of radiation era: **matter-radiation equality**
i.e., $\rho_{\text{matter}} = \rho_{\text{radiation}}$ when $z_{\text{eq}} \sim 3 \times 10^4$
Q: is before or after BBN? recomb?
$\Rightarrow$ this marks onset of structure formation

Q: how does this update our naive CMB calculation?
Hint: then, correct reasoning for $\delta = \delta_b$ only

baryons tightly coupled to photons till recombination
$\rightarrow$ so dark matter perturbations begin growth earlier

And so: DM has grown more! update earlier estimate and focus on dark matter

$$\delta_{m,0} = \frac{D_{\rm ls}}{D_{\rm eq}} \delta_{b,0} \sim \frac{1+z_{\rm eq}}{1+z_{\rm ls}} \delta_b \sim 30 \times 0.05 \sim 1 \qquad (18)$$

DM can grow to nonlinearity today!

★ existence of collapsed cosmic structures *requires* collisionless dark matter!
★ independent argument for large amounts of weakly interacting matter throughout universe!

Inflation and Sachs-Wolfe

Inflation: quantum fluctuations → density fluctuations

- adiabatic (all species)
- Gaussian
- scale invariant–what does this mean?

In detail: inflation predicts that the dimensionless fluctuations in the *gravitational potential* ↔ *local curvature* are independent of scale
→ this was what we really calculated in Inflation discussion

inflationary scale-invariance is for grav potential:
i.e., Fourier mode contribution $\Delta_\Phi^2 \sim k^3 |\Phi_k|^2 \sim const$ indep of k
→ scale invariant: $|\Phi_k|^2 \sim k^{-3}$
Q: how related ot $P(k)$?

need to connect gravitational potential/curvature perturbations to density perturbations

But in Newtonian regime, know how to do this:
Poisson relates potential and density:

$$\nabla^2 \delta\Phi = 4\pi G \delta\rho \quad \rightarrow \quad \Phi_k \sim \delta_k / k^2 \tag{19}$$

and so $P(k) = |\delta_k|^2 \sim k^4 |\Phi_k|^2$

thus scale invariant gravitational potential
gives power spectrum:

$$P_{\mathsf{scale-inv}}(k) \sim k^4 \, |\Phi_{\mathsf{scale-inv}}(k)|^2 \; \sim \; k \tag{20}$$

i.e., scale invariance: $P(k) \sim k^n$, $n_{\mathsf{scale-inv}} = 1$