Astro 507

Lecture 39

May 1, 2020

Announcements:

- **Preflight 6b due today**
- **Preflight 6c due Wednesday**
  give feedback on others

Last time: a tale of two species–dark matter and baryons

Q: why do DM vs baryon perturbations behave differently?

Q: which begins growth first?

dark matter: dominates matter density
weakly interacting $\rightarrow$ treat as pressureless
baryons: (really, both nuclei and electrons) subdominant density
before recombination, feel strong photon pressure

radiation era: perturbation growth suppressed
due to large pressure, high sound speed $c_s = c/\sqrt{3}$
so Jeans length $\sim$ sound horizon $\sim$ particle horizon

matter-dominated era before recombination:

- dark matter fluctuations grow form potential wells
  beginning at matter-radiation equality $z_{\rm eq} = 3402 \pm 26$
- insize horizon: baryons oscillate in DM potentials

after recombination:

- baryons see pressure drop, collapse
  CMB released, today gives snapshot of oscillations

Small Scales: Acoustic Oscillations

at matter-radiation equality:
dark matter has been collapsing
overdensities forms gravity potential wells

baryons begin oscillations:
standing waves driven by DM gravity
• with radiation pressure as restoring force
continues until recombination

scale λ has $\delta_b(k) \sim -\cos(2\pi c_s t/\lambda)$
slowest oscillation is near sound horizon $\lambda \sim r_s = \int c_s dt/a$
give largest-scale δ_b^2 extremum:
at recombination first compression (1/2 period)

Q: what scale gives next δ_b^2 extremum? and the next?

CMB Acoustic Peaks

consider density variance δ_b^2 at different λ
at time of recombination

- $\lambda_{\text{comov}} \sim r_s$: 1/2 cycle completed
  first compression $\rightarrow$maximum δ_b^2
  extra strong since baryon gravity adds to potential
- $\lambda_{\text{comov}} \sim r_s/2$: 1 cycle completed
  first rarefaction: minimum in δ_b but maximum δ_b^2
  weaker than first peak since baryon gravity minimized

Q: in general, what's the pattern of extrema?

Q: on small scales, is an overdensity a hot spot or cold spot? why?

Spots Cold and Hot: Small Scales

Define temperature fluctuation $\Theta = \delta T/T$

On Small Scales: Adiabatic

standing waves lead to fluctuations in $\rho_b \sim T^3$, so

$$\Theta \equiv \frac{\delta T}{T} = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_b \quad (1)$$

$\Rightarrow$ extrema in density $\rightarrow$ extrema in $\Theta \propto \delta_\gamma$

$\star$ photon T contrast reflects T distribution at source

- but both high *and* low density give *large* $(\delta T/T)^2$!
  photon climb out of potential doesn't change $\delta T/T$ much
  $\rightarrow$ CMB hot spots are high density, cold are low

Q: what about on large scales $\lambda > r_s$?

Very Large Scales: Sachs-Wolfe

beyond horizon: no oscillations, main effects gravitational (GR):

- **gravitational redshift:** photon climbs out of potential $\delta\Phi < 0$
  redshift $\delta\lambda/\lambda = \Phi_0 - \Phi_{\rm ls} = -\delta\Phi$
  and since $T \sim 1/\lambda$, $(\delta T/T)_{\rm redshift} = \delta\Phi$: photons cooled!
- **time dilation:** takes longer to climb out of overdensity
  looking at younger, hotter universe
  $\delta t/t = \delta\Phi$, and since $a \sim t^{2/3}$ and $T \sim 1/a$
  then $T \sim t^{-2/3}$, and $(\delta T/T)_{\rm dilation} = -2/3\ \delta\Phi$

net effect: Sachs - Wolfe

$$\left(\frac{\delta T}{T}\right)_{SW} = \left(\frac{\delta T}{T}\right)_{\rm redshift} + \left(\frac{\delta T}{T}\right)_{\rm dilation} = \frac{1}{3}\delta\Phi \qquad (2)$$

⋆ overdensities are cold spots, underdensities hot

Note: this regime is what tests inflation
Q: what predicted?

Inflation and Sachs-Wolfe

Inflation: quantum fluctuations $\rightarrow$ density fluctuations

- adiabatic (all species)
- Gaussian
- scale invariant–what does this mean?

Extras: inflation scale invariance for wavenumber k
sets "power spectrum" – mean-square fluctuation at k

$$\left\langle |\delta_k|^2 \right\rangle = P_{\text{scale-inv}}(k) = Ak^{n_{\text{scale-inv}}} \tag{3}$$

with A a constant (sets fluctutaions amplitudes)
and $n_{\text{scale-inv}} = 1$

Predictions:

- fluctuations occur on all scales
- largest amplitudes for big k $\rightarrow$ small scales
- $\delta_k \rightarrow 0$ for $k \rightarrow 0$, as we must find *Q: why?*

Angular vs Linear Scales

So far: decomposed fluctuations in (3-D) $\vec{k}$-space
but observe on sky: 2-D angular distribution

Transformation: projection of plane waves
at fixed k: see intersection of wave with last scattering shell
`www: Wayne Hu animation`

appears on a range of angular scales
but typical angular size is $\theta \sim \lambda / d_{\text{rec,com}} \sim (k d_{\text{rec,com}})^{-1}$

large angles $\rightarrow$ large λ (check!)
for large angular scales $\theta > \theta_{\text{hor,diam}} \sim 1^\circ$, superhorizon
perturbations not affected by oscillation
for small angular scales, see standing waves

- peaks at extrema, harmonics of sound horizon
  k are in ratios 1:2:3:...

Intermission

The CMB Observed

- Observe 2-D sky distribution of $\frac{\Delta T}{T}(\hat{n}) \equiv \Theta(\hat{n})$ in direction $\hat{n}$
- Decompose into spherical harmonics

$$\Theta(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \tag{4}$$

with Y_{lm} spherical harmonics *Q: why not* $\ell = 0, 1$*?*
Q: angular size vs ℓ*?* λ *vs* ℓ*?*

Form angular correlation function *Q: what is this physically?*

$$\langle \Theta(\hat{n}_1)\Theta(\hat{n}_2) \rangle = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) \left\langle |a_{\ell m}|^2 \right\rangle P_\ell(\hat{n}_1 \cdot \hat{n}_1) \tag{5}$$

$$= \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) C_\ell P_\ell(\cos\vartheta) \tag{6}$$

where $\cos\vartheta = \hat{n}_1 \cdot \hat{n}_1$
Q: averaged over the m *azimuthal modes–why?*

all interesting anisotropy information encoded in

$$C_\ell = \left\langle |a_{\ell m}|^2 \right\rangle \tag{7}$$

isotropy $\rightarrow$ azimuthal dependence averages to zero

Note: analog of Δ^2 (variance per log scale) is
$\mathcal{T}^2(\ell) = \ell(\ell+1)C_\ell$: usually what is plotted

Since $P_\ell(\cos\theta) \sim (\cos\theta)^\ell \sim \cos(\ell\theta)$
at fixed ℓ, angular size $\theta \sim 2\pi/\ell = 180^\circ/\ell$
e.g., $\ell = 2$ quadrupole $\rightarrow \theta \sim 90^\circ$
and horizon size $\theta \sim 1^\circ$ is at $\ell \sim 200$

and since $\theta \sim \lambda/d_{\rm rec} \sim 1/dk$:

multipoles scale as $\ell \sim 1/\theta \sim k \sim 1/\lambda$
low $\ell \rightarrow$ big angular, physical scales $\rightarrow$ small k

CMB Anisotropy Observations: Strategy

- achieve high sensitivity, remove systematics
  make a "difference experiment"
  i.e., measure δT directly, don't subtract
- observe as much of the sky as possible (or as needed!)
  balloons/ground: limited coverage
  satellites (COBE, WMAP, Planck): all-sky
- remove Galactic contamination: "mask" plane
- recover Θ for observed region

- decompose into spherical harmonics $Y_{\ell m}$
- construct power spectrum $\ell(\ell+1)C_\ell$
- report results
- collect thousands of citations, prominent Prizes

CMB Temperature Anisotropies: Results

COBE (1993)

- first detection of $\delta T/T \neq 0$
- receiver horn angular opening $\sim 8^\circ$
  $\rightarrow$ only sensitive to large angular scales
  i.e., superhorizon size
- found $(\delta T/T)_{\rm rms} \sim 10^{-5}$
- power $\ell(\ell+1)C_\ell$ flat $\rightarrow$ implies $P(k) \sim k$!
  $n = 1$ spectrum: scale invariant!

Interregnum (late 90's, early 00's)

- ground-based, balloons confirmed anisotropy
- acoustic peaks discovered
  strong indication of first peak

WMAP (2003-)

- first all-sky survey of small angular scales
- $n = 1$ confirmed, indication of small tilt $n - 1 \neq 0$?
  consistent with inflation! and non-trivially so!
- acoustic peaks mapped: good measurement of 1st, 2nd
  detection of third
- first peak: $\ell \sim 200$ horizon at recomb!
- power dropoff seen at large ℓ
  $\rightarrow$ nonzero thickness of last scattering
  due to photon diffusion, non-instantaneous decoupling

Planck (2013–2018)

- exquisite measurements of cosmological parameters
- small deviation from scale invariance measured at high confidence
- no detection of non-Gaussianity
- sensitivity to N_{eff} and Y_{p}

CMB Polarization

Recall: pre-recombination, photons coupled to baryons
via Thompson scattering with electrons

Key fact: Thompson scattering is anisotropic and polarized
polarization of scattered radiation scales as

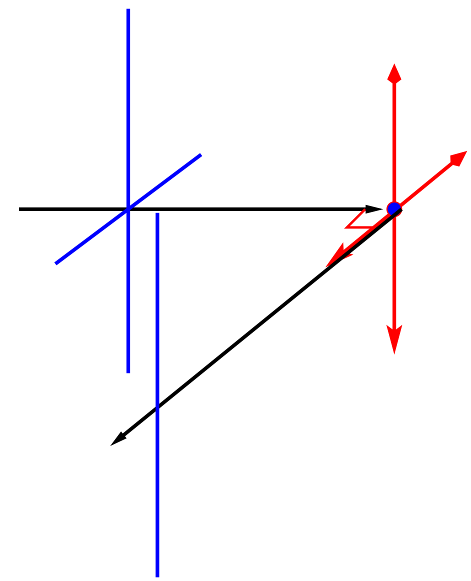
$$\frac{d\sigma_T}{d\Omega} \propto |\hat{\epsilon}_{\rm in} \cdot \hat{\epsilon}_{\rm sc}|^2 = \cos^2\theta \tag{8}$$

where:
$\hat{\epsilon}_{\rm in}$ is *incident photon polarization*
$\hat{\epsilon}_{\rm sc}$ is *scattered photon polarization*
and propagation is transverse: $\hat{\epsilon}_{\rm in} \cdot \hat{n}_{\rm in} = 0$

Q: in what direction is polarization max? min?
Q: why physically? hint: think of e as antenna

max scattered polarization when in plane normal to initial pol'n
zero scattered intensity in direction of initial pol'n

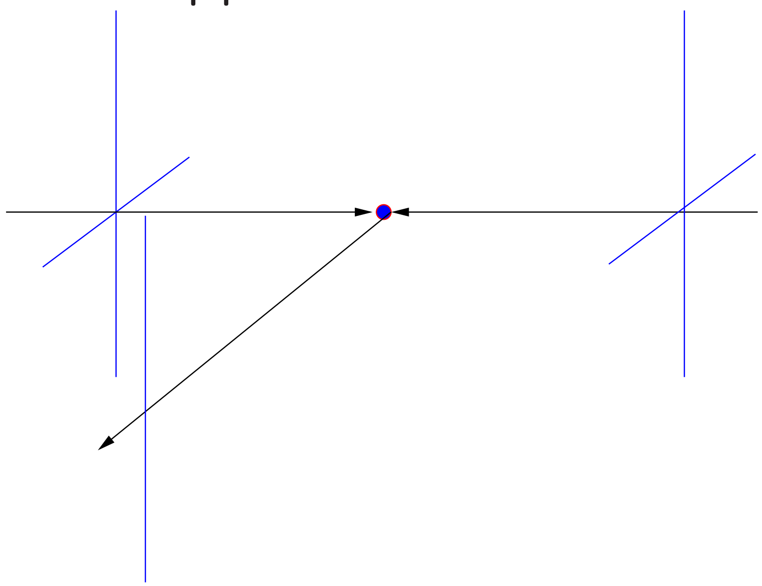
classical picture: e^- as dipole antenna
incident polarized wave accelerates e^-
$\rightarrow$ azimuthally symmetric radiation,
peaks in $\theta = 0$ plane



note: since $\cos^2\theta \propto \cos 2\theta$, scattered rad has 180^0 periodicity
$\rightarrow$ a "pole" every 90^0: quadrupole

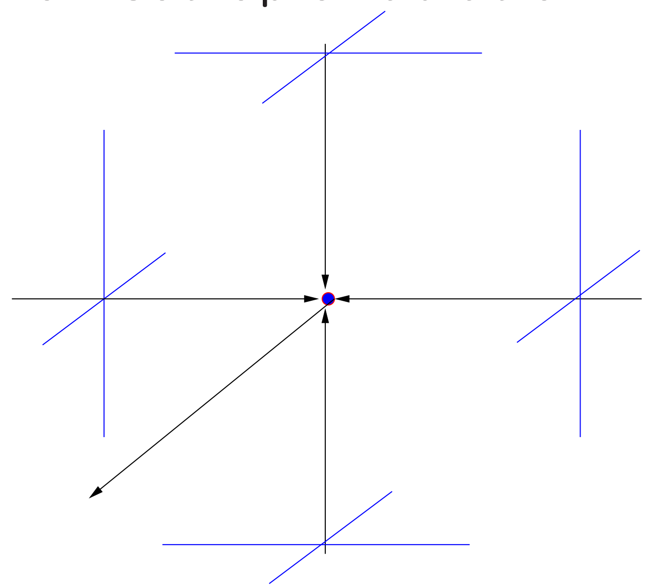
Q: what if unpolarized radiation from 2 opposite directions?
Q: what if isotropic unpolarized radiation?

from opposite incident directions:



still linearly polarized!

for isotropic radiation:



unpolarized!

...as demanded by symmetry

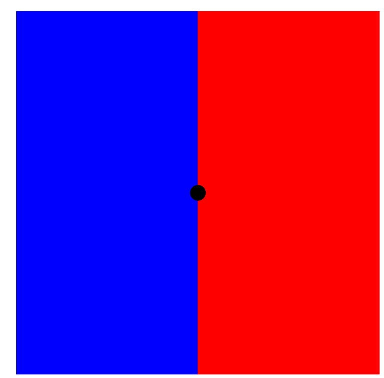
Polarization and Inhomogeneity

Pre-recomb: repeated Thompson scattering
randomizes polarization → CMB unpolarized

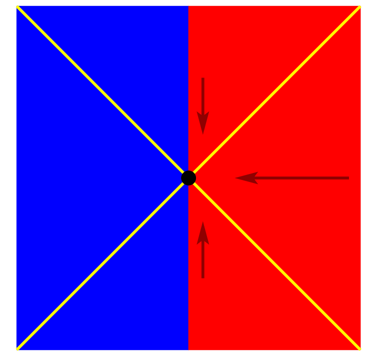
But at recomb, last scattering evens "uncompensated"

- if plasma homogeneous: still no net polarization
- but inhomogeneities → net linear polarization in CMB

Consider point on hot-cold "wall"
Q: what is scattered polarization? why?
Q: what temperature pattern seen at point?



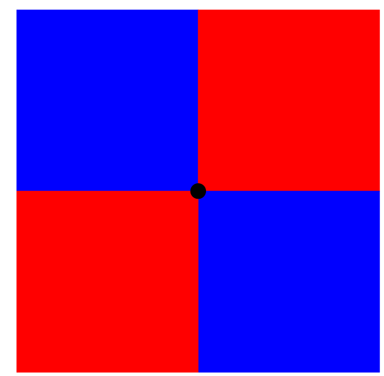
dipole anisotropy:
unpolarized



pattern seen at point:

dipole anisotropy

extra polarized radiation from hot region cancels

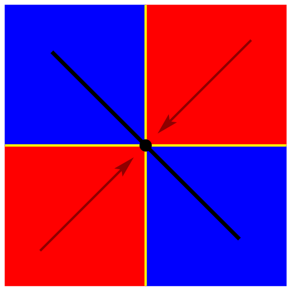


Now consider point on “ checkerboard vertex”

Q: what is scattered polarization? why?

Q: what temperature pattern seen at point?

point sees *quadrupole* anisotropy
extra polarization from hot regions
doesn't cancel



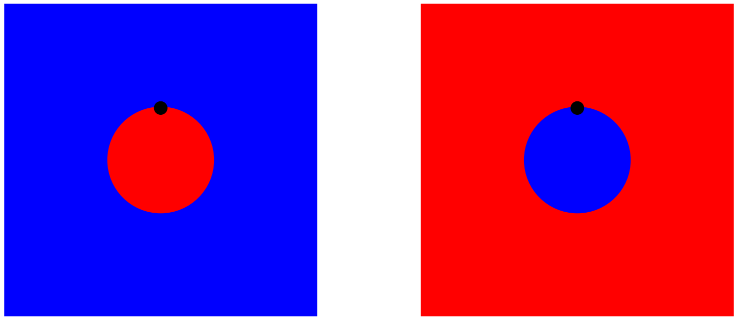
quarupole anisotropy:
linear polarization

→ net linear polarization towards us, aligned w/ "cold" axis

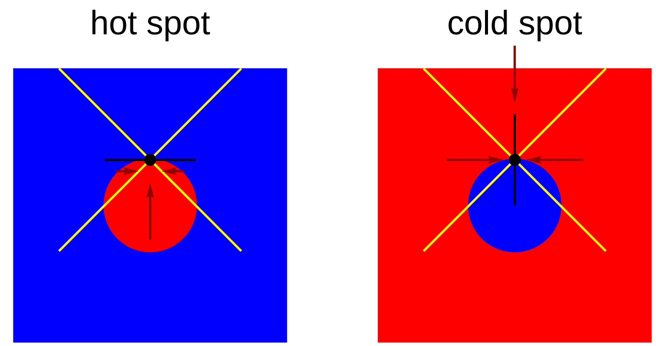
`www: cool Wayne Hu movie`

Q: what about edge of circular hot spot? cold spot?

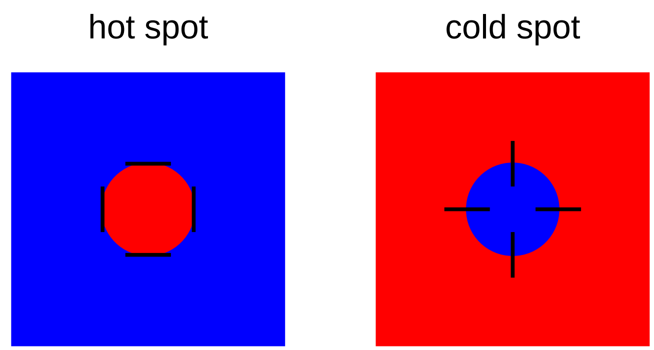
hot spot cold spot



at a single point on edge:



so by symmetry:



polarization tangential (ring) around hot spots
radial (spokes) around cold spots
(superpose to "+" = zero net polarization–check!)

`www: WMAP polarization observations of hot and cold spots`

Note: polarization & T anisotropies *linked*
$\rightarrow$ consistency test for CMB theory and hence hot big bang

Director's Cut Extras

Recombination: Snapshot Taken

At recombination, free e^- abundance drops

baryons quickly decouple from photons

huge drop in pressure $\rightarrow c_s \rightarrow 0$

begin to collapse onto DM potentials

photons travel freely (typically) afterwards

fluctuation pattern at recomb is "frozen in"

δ vs scale records different # of cycles at recomb

$$P(k) = \|\delta_k\|^2 \sim \frac{\sin(2kc_s\eta_{\text{rec}})}{2kc_s\eta_{\text{rec}}} P_{\text{init}}(k) \tag{9}$$

written onto temperature pattern ("say cheese!")

Recomb fast $\rightarrow$ CMB is image of last scattering surface

Pre-Recombination: Acoustic Oscillations

Baryons in DM-dominated background

$$\ddot{\delta}_b + 2\frac{\dot{a}}{a}\dot{\delta}_b \simeq 4\pi G\rho\delta_{dm} - \frac{k^2c_s^2}{a^2}\delta_b \sim \frac{\delta_{dm}}{t^2} - \frac{k^2c_s^2}{a^2}\delta_b \tag{10}$$

key comparison: mode scale $\lambda \sim k^{-1}$
vs **comoving sound horizon** $c_st/a = d_{s,com}$

for *large scales* $kc_st/a \ll 1$: *baryons follow DM*
for *small scales* $kc_st/a \gg 1$: *baryons oscillate*, as

$$\delta_b \sim \frac{1}{\sqrt{ac_sk}} e^{i\int kc_s d\eta} \tag{11}$$

(PS 6) where $d\eta = dt/a$ is conformal time

Q: for fixed k, what is δ time behavior?
Q: at fixed t, what is δ pattern vs k?
Q: what sets largest λ that oscillates?

baryonic perturbations do not grow, but oscillate:

$$\delta_b \sim \frac{1}{\sqrt{a c_s k}} e^{i \int k c_s d\eta} \tag{12}$$

to simplify, imagine constant c_s, $\delta_b \sim e^{ikc_s\eta}$

at fixed k, sinusoidal oscillations

phase counts number of cycles $N = kc_s\eta/2\pi = c_s\eta/\lambda$

oscillation frequency: $\omega \sim kc_s \sim c_s/\lambda \propto 1/\lambda$

at fixed $t \rightarrow$ fixed η:

small λ and large $k \rightarrow$ rapid oscillations

largest oscillations at scale $\lambda \sim c_s\eta \sim c_s t/a$: *sound horizon*

Q: when do oscillations stop? observable signature?

CMB Anisotropies and Cosmological Parameters

Small angular scales: peaks at density extrema
can measure peak scales (ℓ positions), amplitudes

Peak Positions

recall: all oscillations begin together (in phase)
then scale k has phase $\omega\eta = c_s k\eta$
observe: density at recomb, when phase is $c_s k d_{\rm rec}$

Q: what is largest scale to show δT extremum? what extremum?
Q: what is next largest scale to show extremum?
Q: implications?

oscillations begin together (in phase)
at recomb, scale k has phase $c_s k d_{\text{rec}}$

peaks at extrema
1st peak: scale that just reached 1st compression
2nd peak: scale that just reach 1st rarefaction
3rd peak: scale that just reach 2nd compression...

peak locations: comoving wavelengths

$$\lambda = 2 c_s d_{\text{rec,com}} \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right) \tag{13}$$

harmonics, first peak scale $\sim 2 d_{\text{s,hor}} \sim d_{\text{hor,com}}$

Q: what do we actually measure on CMB sky? implications?

fluctuation size on sky = angular diameter measurement!
standard ruler=comoving horizon
sensitive to geometry → curvature
⇒ peak positions: Ω_0

why? in flat matter-dominated U: physical particle horizon is

$d_{\rm hor,phys}(z) = (1+z) d_{\rm hor,com}(z) = (1_z) \int_0 dt/a 2\Omega_{\rm m}^{-1/2} d_{\rm H,0}(1+z)^{1/2}$

angular diameter distance is

$$d_{\rm A}(z) = \frac{r(z)}{1+z} = 2\Omega_{\rm m}^{-1/2} d_{\rm H,0}(1+z) \qquad (14)$$

and so expect *sound horizon* angular diameter

$$\vartheta_{\rm hor,s} = \frac{c_s}{c}\vartheta_{\rm hor} = \frac{c_s}{c}\frac{d_{\rm hor,phys}(z_{\rm rec})}{d_{\rm A}(z_{\rm rec})} \simeq \frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+z_{\rm rec}}} \sim 1^\circ \qquad (15)$$

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WMAP: first peak at $\ell_{\rm peak} \sim 200 \to \vartheta_{\rm peak,obs} \sim 1^\circ \to$ $\Omega_0 = 1$
Q: what about amplitudes? 1st peak? 2nd peak?

Acoustic Peak Amplitudes

Amplitude measures degree of compression/rarefaction
$\rightarrow$ strength of driving force $\rightarrow$ matter density
mostly DM density, but baryons too

Q: effect of baryons on 1st peak? 2nd peak? other peaks?

Effect of baryons: alter the gravitational potential well

▷ during compression: baryons make well deeper

▷ during rarefaction: baryons make well shallower

Net effect: higher $\Omega_{\text{baryon}} \rightarrow$ bigger odd peaks (compression)

smaller even (rarefaction) peaks

If measure one of each, e.g., 1st + 2nd peaks $\rightarrow$ get Ω_B!

⋆ CMB is cosmic baryometer!

⋆ independent of BBN (also more precise)

As we saw: decent CMB-BBN concordance

...but Li problem remains

Q: why do fluctuations die off at small angular scales?

CMB Damping Tail

so far: assumed recombination *instantaneous*

- decoupling as sharp transition
- all photons have last scattering at same instant

in reality:
last scattering is smooth transition
photon mean free path quickly but smoothly increases
$\rightarrow$ not all photons last scatter at same time
$\rightarrow$ *last scattering surface has nonzero thickness*

expect CMB signal *damping* on scales $\ll$ thickness
$\rightarrow$ on smaller scales, photon scattering diffusive
$\rightarrow$ exponentially suppresses temperature signal