Astro 507

Lecture 4

Jan 29, 2020

Announcements:

- Preflight 1 due Friday at noon
  be sure to do both parts: discussion + reading response

Today:

⋆ Implications of Cosmo Principle + Hubble Law

⋆ Cosmodynamics I–Newtonian Cosmology

Don't Be Perturbed ... Yet

The Universe is rich and complex
our understanding will come from a series of ever-better and ever more complex *approximations*

for the beginning of the course:
"*zeroth order*" approximation: the "*unperturbed universe*"
$\Rightarrow$ study universes where *cosmological principle is exact*
- perfect homogeneity and isotropy
- the only bulk motions are due to "Hubble flow"

by the end of the course:
higher order approximation: include density perturbations
- how we think they were created
- how they grow over time
- how they affect cosmic structure and dynamics

Recap

Last time:
Observational/Conceptual Foundations of Cosmology

⋆ Cosmological Principle *Q: namely?*

⋆ Observed Cosmic Kinematics: Hubble's Law
Q: what's that?
Q: what's a parameter, what's a variable?
Q: qualitative behavior–restate in simple language?
Q: velocity field pattern?

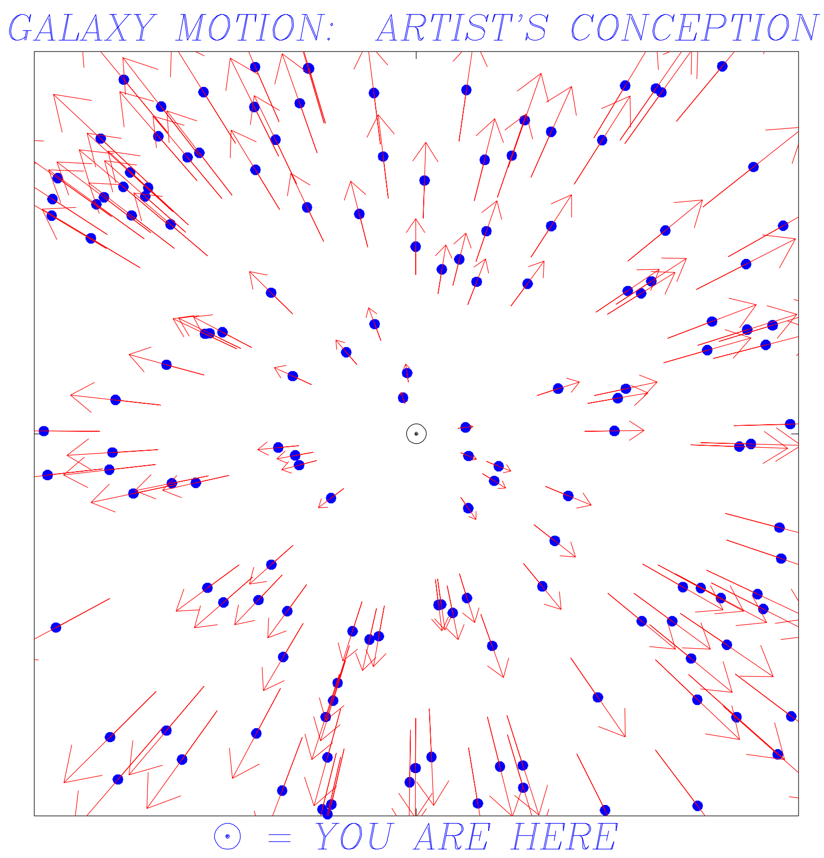
Structure + Dynamics: Evolution

observe:

- U. homogeneous, isotropic
- Hubble law $\vec{v} = H\vec{r}$

this pattern cries out for explanation!

a consistent model must
account for both properties



Q: how to reconcile?
at least 2 logical possibilities...

Explaining Hubble: Kinematic Model of Milne (1933)

imagine an explosion at $t = 0$

- let galaxies all start in region of size $\ll ct_{\mathrm{today}}$
  fly away with uniform distribution speeds v_{gal}
- we remain at center $r = 0$ until now: $t_{\mathrm{today}} = t_0$

after explosion, let each galaxy *coast*
maintaining its initial velocity

after time $\Delta t = t_0$:

- $\vec{r}_{\mathrm{gal}} \to \vec{v}_{\mathrm{gal}}\ t_0$ fastest $\to$ farthest!
- so $\vec{v}_{\mathrm{gal}} \to \vec{r}_{\mathrm{gal}}/t_0 \equiv H_0 \vec{r}_{\mathrm{gal}} \propto \vec{r}_{\mathrm{gal}}$: recover Hubble's law!
- solve for cosmic age

$$t_{0,\mathrm{Milne}} = \frac{1}{H_0} \tag{1}$$

Hubble Flow: Characteristic Scales

Hubble's law today: $\vec{v} = H_0 \vec{r}$
introduces Hubble parameter $H_0 = 100\ h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$
with $h \approx 0.7$

Hubble time

$$t_{\mathrm{H}} \equiv \frac{1}{H_0} = 9.778\ h^{-1}\ \mathrm{Gyr} = 13.97\ \mathrm{Gyr} \left(\frac{0.70}{h}\right)$$

where 1 Gyr $= 10^9$ years
$\Rightarrow$ sets $\sim$ scale of "expansion age" of Universe

Hubble length

$$d_{\mathrm{H}} \equiv \frac{c}{H_0} = c t_{\mathrm{H}} = 2.998\ h^{-1}\ \mathrm{Gpc} = 4.283\ \mathrm{Gpc} \left(\frac{0.70}{h}\right) \qquad (2)$$

a useful lengthscale even to non-egoists!

Q: why? what does this measure?

The Hubble Length

In the Milne Kinematic/Explosion Model

- c sets maximum galaxy speed
- so max distance travelled is $d_{\mathrm{max}} = ct_0 = d_{\mathrm{H}}$
  size of Milne universe: distance to edge!
- but also: d_{H} is max range of photons
  limit to size of the Universe we can see

Beyond Milne

- homogeneity: no center, no edge!
- but finite lightspeed still important

⋆ Hubble length d_{H} sets scale of *observable* Universe

⋆ and thus gives scale of causally connected Universe

Kinematic/Egoist/Explosion Model (Milne) is logically possible!
i.e., can fit basic cosmo structure, kinematic data

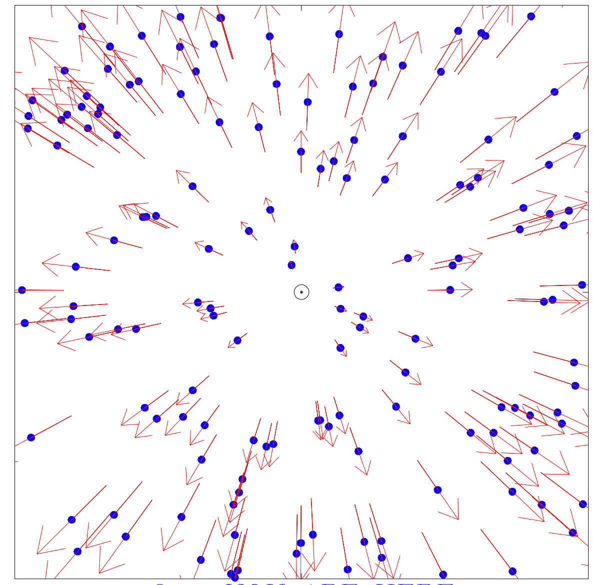
But...

Q: give a philosophical reason why we don't believe this?

Q: give a physical reason why this treatment can't be right?

Q: give an observational reason why we don't believe this?

Critiques of Cosmic Egoism



Are we at the center of the universe?

Philosophically:

• *not Copernican* ("principle of mediocrity")

Physically:

• *haven't included gravity!*

Observationally:

• *Milky Way, Local Group don't look special*

not what expect from center of explosion

cf. supernova $\rightarrow$ compact remnant: neutron star, black hole

...yet radial v pattern does make us *look* special...

The Magic of Hubble

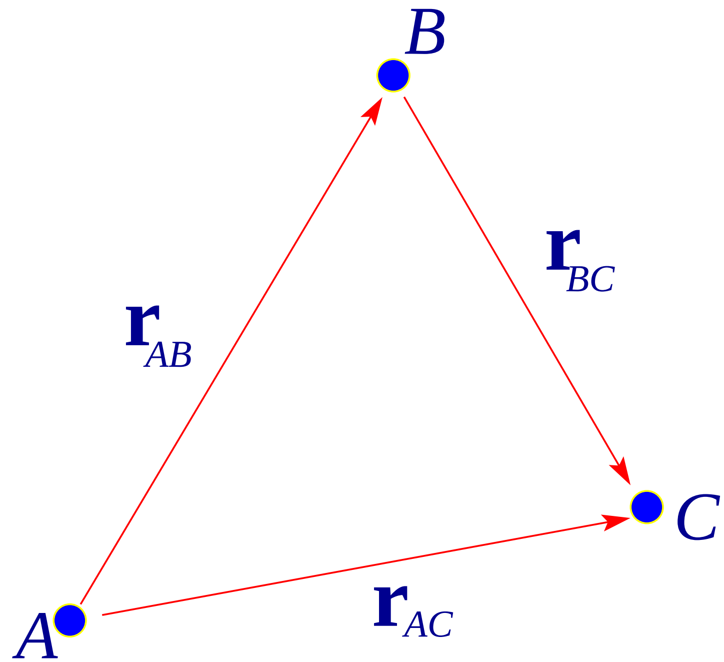
consider three arbitrary cosmic points:

$\vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB}$

Assume A sees Hubble's law:

- $\vec{v}_{AB} = H\vec{r}_{AB}$
- $\vec{v}_{AC} = H\vec{r}_{AC}$

Then ask: *what does B see? C?*



find velocities relative to B:

$\vec{v}_{BC} = \vec{v}_{AC} - \vec{v}_{AB} = H(\vec{r}_{AC} - \vec{r}_{AB}) = H\vec{r}_{BC}$

This is huge!

Q: why? What have we proven?

we have shown:

if A sees Hubble's law, then so do (arbitrary) B and C

thus: if *any* observer measures Hubble's law

then *all* observers will measure Hubble's law!

so: Hubble law implies

$\rightarrow$ *all* galaxies recede according to same law

$\rightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion

which preserves homogeneity and isotropy

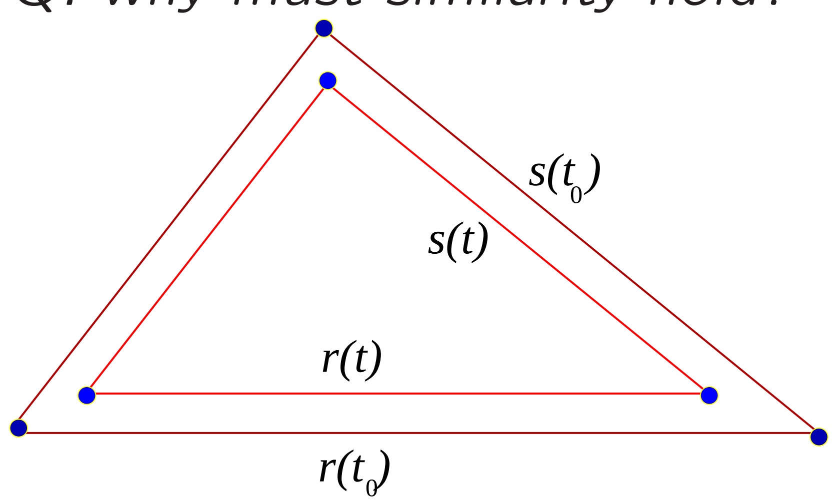
i.e., *any* other motion breaks cosmo principle

...and lo, Hubble law is exactly the observed motion!

Cosmo Principle Constrains Kinematics

consider arbitrary triangle defined by 3 observers at t
Hubble law $\rightarrow$ observers in relative motion
$\rightarrow$ at later time t_0, larger triangle

the claim:
later Δ always *similar to* original Δ
Q: what are similar triangles?
Q: why must similarity hold?



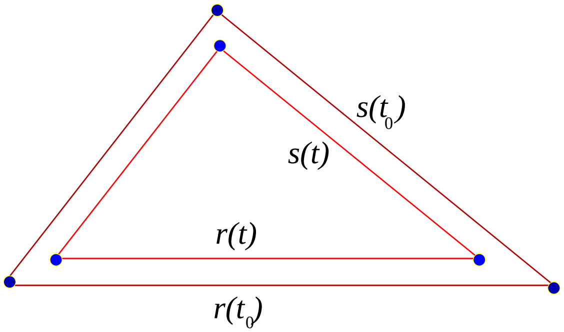
Q: connections among r's and s's?

similar: triangle angles preserved

$\Rightarrow$ *side ratios preserved*,

so ratios at any two time

$$\frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$



holds for any triangle,

so side *length ratio depends only on time t:*

$$a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$

Q: what does this imply about cosmic kinematics?

We have shown:
Cosmo Principle demands *any length* $r(t)$ evolves as

$$r(t) \propto a(t) \tag{3}$$

and so without loss of generality we may write

$$\boxed{r(t) = a(t)\ r_0} \tag{4}$$

where we choose $\boxed{a(t_0) = 1 \text{ today}}$, and
$r_0 = r(t_0)$ is *present value* of length ("comoving coordinate")

$a(t)$ must be universal **cosmic scale factor**
can depend only on time
and at any t: a has same value everywhere in space

This is huge!
Q: why? What have we proven? What is character of motion?

Explaining Hubble: Expansion

the meaning of Hubble Law: Take 2

2. Einstein interpretation:
using General Relativity:
Universe is expanding
all galaxies receding from all others
bold, strange idea!

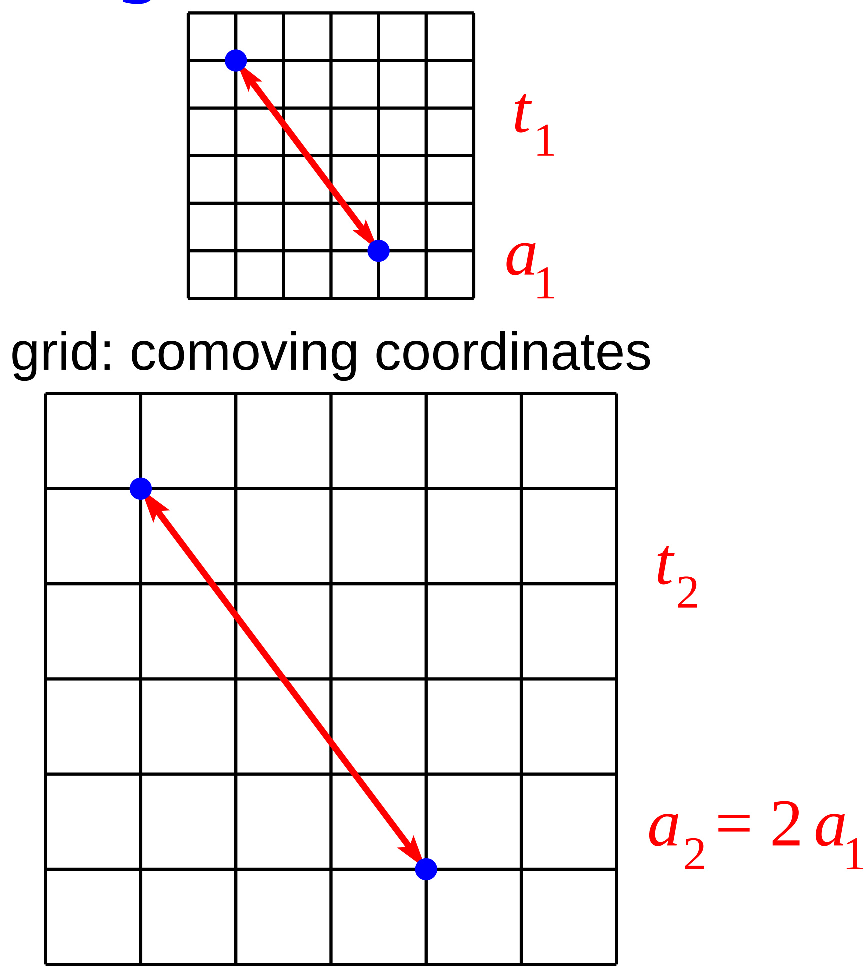
In fact: Einstein himself initially found it unacceptably strange
in 1917, modified GR equations with "fudge factor"
$\rightarrow$ "cosmological constant" Λ designed to keep Universe static
after Hubble's 1929 work, Einstein allegedly said this was
his "greatest blunder"
...but wait a few lectures...

Expansion: Stretching Coordinate Grid

comoving grid

with two comoving observers

factor 2 expansion



Expansion: Einstein $\to$ Hubble

`www: animation demo: cosmic expansion`

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances

$$\vec{r}(t) = a(t)\ \vec{r}_0$$

so velocity is: where "overdot" is time derivative: $\dot{x} \equiv dx/dt$

$$\vec{v}(t) = \dot{\vec{r}} = \vec{r}_0\ \dot{a} = \frac{\dot{a}}{a}\ a\ \vec{r}_0 \equiv H(t)\ \vec{r}(t) \qquad (5)$$

$\Rightarrow$ recover Hubble law!
now interpret "Hubble parameter"
as **expansion rate** $H(t) \equiv \dot{a}/a$

Cosmic Scale Factor Revisited

for two "particles" (possibly Galaxies!)
distance evolves according to

$$\vec{\ell}(t) = \underbrace{a(t)}_{\substack{\text{scale factor} \\ \textit{time varying}}} \quad \underbrace{\vec{\ell}_0}_{\substack{\text{present distance} \\ \textit{fixed once and for all}}} \tag{6}$$

and thus

$$\vec{v} = H\vec{\ell} \tag{7}$$

with $H = \dot{a}/a$

Q: implications–present, past, future values for a?

present: at t_0, $a(t) = 1$ by convention
Universe is expanding, so

past: $a(t) < 1$
future: $a(t) > 1$

e.g., at some time in past $a = 1/2$
"galaxies twice as close"

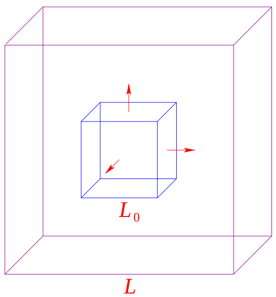
Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners

present side length L_0

at any time: length $L(t) = a(t)\, L_0$



cube is "comoving" w/ expansion

- *volume* $V = L^3 = L_0^3 a^3 = V_0 \; a^3$, thus $V \propto a^3$
- area of a side: $A = L^2 = A_0 \; a^2$, thus $A \propto a^2$

`www: raisin cake analogy`

`www: balloon analogy`

Q: what is tricky, imperfect about each analogy?

Cosmodynamics II

$a(t)$ gives expansion history of the Universe
which in turn tells how densities, temperatures change
$\rightarrow$ given $a(t)$ can recover all of cosmic history!

but...

How do we know $a(t)$?
What controls how scale factor $a(t)$ grow with time?

Q: what force(s) are at work microscopically? between galaxies?
Q: how are the force(s) properly described?