Astro 507

Lecture 40

May 4, 2020

Announcements:

- **Preflight 6c due Wednesday**

  give feedback on others

  thanks for your nice contributions

Last time: baryons, dark matter, and the CMB

Q: pre-recombination behavior on small scales: DM vs plasma?

Q: CMB signature on small scales?

Q: CMB signature on large scales?

The CMB Observed

- Observe 2-D sky distribution of $T(\hat{n}) \equiv \Theta(\hat{n})$ in direction $\hat{n}$
- Decompose temperature into spherical harmonics

$$T(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \; Y_{\ell m}(\theta, \phi) \tag{1}$$

with $Y_{\rm lm}$ spherical harmonics, and coeffiencients

$$a_{\ell m} = \int T(\theta, \phi) \; Y_{\ell m}(\theta, \phi) \; d\Omega \tag{2}$$

- Lowest mode: $\ell = 0$, $m = 0$, $Y_{00} = 1$
  so $a_{00} = \int T_{\rm obs}(\theta, \phi) \; d\Omega = \langle T \rangle$: *monopole*
- Next: $\ell = 1$, $m = (-1, 0, +1)$, $Y_{1m} = e^{im} \cos\theta$
  so $a_{lm} = \int T(\theta, \phi) e^{im} \cos\theta \; d\Omega = \langle T \rangle$: *dipole*
  note: cos term automatically removes mean
  isolates fluctuation, here on angular scales 180°
  same goes for higher order terms ($\ell = 2$ quadrupole, etc)

Angular Correlation Function

Spherical harmonics are "Fourier transform of sky"

Construct 2-point **angular correlation function:**
compare T at two directions separated by angle θ

$$\langle T(\hat{n}_1)\ T(\hat{n}_2)\rangle = \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)\left\langle |a_{\ell m}|^2 \right\rangle P_\ell(\hat{n}_1\cdot\hat{n}_1) \quad (3)$$

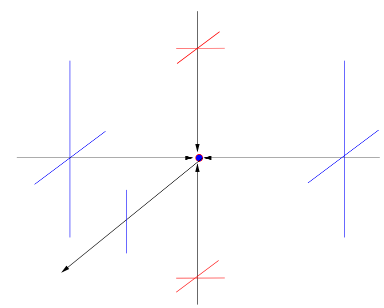
$$= \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)C_\ell P_\ell(\cos\vartheta) \quad (4)$$

where $\cos\vartheta = \hat{n}_1\cdot\hat{n}_1$, and P_ℓ is Legendre
roughly: multipole ℓ corresponds to $\theta \sim 180^\circ/\ell$

average over the m azimiuhal modes:
corresponds to no special directions, i.e., global isotropy
$\rightarrow$ all that matters is angle of displacement, not its orientation
this is a testable and verified assumption!

CMB Polarization: Twitter Version

- Thomson scattering $\gamma e \to \gamma e$
  creates polarized photons
  if incident quadrupole anisotropy
- gravitational waves drive quadrupole motion
  and thus are also polarization sources
- as CMB photons pass through cosmic structure
  gravitational lensing also creates polarization



Therefore:

- CMB should have nonzero polarization
- polarization and temperature fluctuations intimately linked
- encodes gravity wave and structure info

Polarization Observed

First detection: pre-WMAP!
⋆ DASI (2002) ground-based interferometer
at level predicted based on T anisotropies! Woo hoo!

WMAP (2003): first polarization-T correlation function

WMAP (2006):
- better statistics
- also polarization autocorrelation

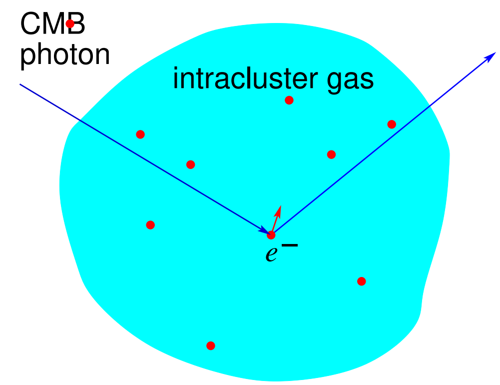
⋆ used T-pol'n links to get model-independent
3-D density power spectrum: consistent with scale invariant!

Today: Planck (2018), SPT etc
- strong detection of polarization, in both E modes and B modes
- temperature–polarization correlations verify basic CMB physics
- clear evidence of B-modes due to gravitational lensing as CMB photons pass through cosmic structure

Sunyaev-Zel'dovich Effect: Twitter Version

the post-recombination Universe is a CMB "foreground"
and does more than lensing

Sunyaev & Zel'dovich (1972):
consider a galaxy cluster
filled with hot, ionized gas
hot intracluster electrons will scatter
a fraction $\tau \sim few\%$ of CMB photons



Q: consequence of $T_{\rm CMB} \ll T_e$?
Q: effect on scattered photons?
Q: cluster CMB image at low ν? high ν?
Q: how to confirm the effect?
Q: what can we do with this?

SZ Effect: A Cosmological Goldmine

becuase $T_{\rm CMB} \ll T_e$: CMB photons gain energy
scattered photons: frequency shift upwards

- at small ν: fewer photons, cluster appears as "hole"
- at large ν: more photons, cluster appears point source
- and there is a null frequency with no change!

Observational signature: check this frequency pattern!
Confirmed! `www: SPT image with SZ holes`

Applications of SZ:

- identification of galaxy clusters independent of redshift! gives unbiased cluster sample!
- probes cluster properties
- probes structure formation

CMB Summary and Outlook

What has the CMB done for us?

- confirmed hot, dense, smooth early universe
- measured primordial power spectrum, consistent w/ inflation
- seen acoustic peaks
- measured a wealth of cosmological parameters
- seen polarization: confirms underlying physics model
- not yet seen: polarization due to primordial gravity waves

What will the CMB do for us?

- very soon (this year and next):
  confirmation(?) of gravity wave signal from inflation!
- CMB as background illumination for structure formation
  SZ effect, 21-cm, ...
- stay tuned!

Intermission: Questions?

Dark Matter–Cold and Hot

Perturbation *growth* & clustering depends on dark matter
internal motions–i.e., "temperature" or *velocity dispersion*
key idea: velocity dispersion (spread) is like pressure
$\rightarrow$ stability criterion is Jeans-like

Cold Dark Matter (CDM)

slow velocity dispersion–trapped by gravitational potentials
no lower (well, very small) limit to structure sizes
perturbation growth only limited by onset of matter dom
$\rightarrow$ small, subhorizon objects form first, then larger
$\rightarrow$ **hierarchical structure formation**: "bottom-up"

Hot Dark Matter (HDM)

high velocity dispersion–escape small potentials
small objects can't form–large must come first
then fragment to form smaller: "top down"

Q: particle candidate for HDM?

Q: physical implications for HDM structure formation?

Q: how can this be tested?

Q: how does HDM alter the power spectrum (transfer function)?

Hot Dark Matter: Neutrino Cocktail

HDM classic candidate: massive ($m_\nu \sim 1$ eV) neutrinos
if light enough, relativistic before $z_{\rm eq}$
→ "free streaming" motion out of high-density regions
→ characteristic streaming scale: horizon size when $\nu \to$ nonrel

$$\lambda_{{\rm FS},\nu} \sim 40\ \Omega_m^{-1/2}\ \sqrt{1\ {\rm eV}/m_\nu}\ {\rm Mpc} \tag{5}$$

★ perturbations on scales $\lambda < \lambda_{\rm FS}$ suppressed
★ $\lambda_{{\rm FS},\nu}$ sensitive to absolute ν masses!

If HDM is dominant DM: expect *no* structure below $\lambda_{\rm FS}$
→ a pure HDM universe already ruled out!

If "mixed dark matter," dominant CDM, with "sprinkle" of HDM
HDM reduces structure below $\lambda_{\rm FS}$
→ $\lambda_{\rm FS}$ written onto power spectrum (transfer function)
→ accurate measurements of, e.g., $P(k)$ sensitive to m_ν
cosmic structure can weigh neutrinos! (goal of DES, et al)

ΛCDM

"Standard" Cosmology today: ΛCDM ...namely:

- FLRW universe
- today dominated by cosmological constant $\Lambda \neq 0$
- with cold dark matter
  $\Rightarrow$ hierarchical, bottom-up structure formation
- ...and usually also inflation: scale invariant, Gaussian, adiabatic

This is the "standard" model but not the only one
Q: arguments in favor?
Q: arguments for other possibilities?
Q: which pieces most solid? which shakiest?

At minimum: ΛCDM is *fiducial* / *benchmark* model
standard of comparison for alternatives

...and so we will adopt ΛCDM the rest of the way

Nonlinear Reality

So far: much success in understanding structures in the linear regime $\delta \ll 1$

But the real universe is nonlinear!
What happens when perturbations become large?
$\Rightarrow$ both theory and observations become challenging!

Theory: nonlinear dynamics rich = interesting = hard
some ingenious analytical approximations, special cases
but serious calculations require numerical solution

Observation: collapsed objects can be easy to find
e.g., bright galaxies–but more to the picture than meets the eye

- can't see the DM halos (usually!); mass doesn't trace light
- how to define a halo? measure its mass?

Q: why would this be ambiguous?

Press-Schechter Analysis

Outlook
adopt hierarchical picture (i.e., some form of CDM)
$\Rightarrow$ matter at *every* point belongs to some structure
over time: go from many small structures to fewer, larger ones

Goal
Given properties of density field–i.e., $P_{\text{init}}(k) = \left\langle \delta_k^2 \right\rangle$
Compute distribution of structures as function of mass, time

Quantitatively: want "mass function"
comoving number density of structures
in mass range $(M, M + dM)$:

$$\frac{dn_{\text{com}}}{dM}(M,t) \tag{6}$$

from this, can compute many other things
e.g., density in $(M, M + dM)$ *Q: which is...?*

Press-Schechter Ingredients/Assumptions

• given mass M, *filter* density field
on comoving length R such that $M = 4\pi/3\ \rho_{\rm bg,com}(t) R^3$
density contrast has *variance* $\sigma^2(M) = \int P(k)\ W(k;R)\ d^3k$

• *in linear regime,* density field obeys *Gaussian statistics*:
in filtered field, probability of finding contrast in $(\delta_{\rm lin}, \delta_{\rm lin} + d\delta_{\rm lin})$:

$$P(\delta_{\rm lin}; M, t)\ d\delta_{\rm lin} = \frac{1}{\sqrt{2\pi\sigma^2(M,t)}} \exp\left[-\frac{\delta_{\rm lin}^2}{2\sigma^2(M,t)}\right]\ d\delta_{\rm lin} \qquad (7)$$

why only good in linear regime *Q: why?*

• Spherical collapse model maps from linear $\rightarrow$ nonlinear
identifies *linear contrast threshold* $\delta_c \simeq 1.69$ for collapsed objects
note: δ_c is time indep! (in EdS cosmo)
$\Rightarrow$ can find fraction of cosmic mass in objects of mass M
Q: how?

fraction of mass or of comoving volume
in collapsed objects of mass M at time t is

$$f(>\delta_c; M, t) = \int_{\delta_c}^{\infty} P(\delta_{\rm lin}; M, t)\ d\delta_{\rm lin} \quad (8)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2(M,t)}} \int_{\delta_c}^{\infty} \exp\left[-\frac{\delta_{\rm lin}^2}{2\sigma^2(M,t)}\right]\ d\delta_{\rm lin} \quad (9)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\delta_c/\sqrt{2}\sigma}^{\infty} e^{-u^2} \equiv \frac{1}{2}{\rm erfc}\left[\frac{\delta_c}{\sqrt{2}\sigma(M,t)}\right] \quad (10)$$

- for realistic $P(k)$, $\sigma^2(M) \sim \int k^3 P(k) W_k(M) dk/k \sim M^{-(n+3)/3}$
  $\rightarrow$ at fixed mass, $\sigma(M,t)$ monotonically decreases with M
  (down to some minimum M cutoff)
- $\sigma(M,t)$ evolves (linearly) as $\sigma \sim a(t) \sim 1/(1+z)$

Q: implications for mass distribution at fixed time?
Q: implications for structure formation over time?

Press-Schechter: mass fraction and structure formation

$$f(>\delta_c; M, t) = \frac{1}{\sqrt{2\pi}} \int_{\delta_c/\sqrt{2}\sigma}^{\infty} e^{-u^2} = \frac{1}{2}\text{erfc}\left[\frac{\delta_c}{\sqrt{2}\sigma(M,t)}\right] \qquad (11)$$

⋆ mass distribution at fixed t:
as filter mass M decreases, variance $\sigma(M)$ increases
▷ more large fluctuations $\rightarrow$ more above threshold
▷ more structures at smaller masses
i.e., $\delta_c/\sqrt{2}\sigma(M)$ decreases $\rightarrow$ larger f
$\Rightarrow$ smallest halos most numerous $\rightarrow$ hierarchy of masses!

⋆ time evolution at fixed M:
at time, scale factor increases, variance $\sigma(t) \propto a(t)$ increases
▷ more structures at fixed mass
▷ small structures merge $\rightarrow$ larger (at expense of smallest)
$\Rightarrow$ hierarchical clustering!

Press-Schechter Mass Function I: Quick-n-Dirty

Press & Schechter (1974):
note that structures can only be made from *over*densities
but *under*densities (voids) occupy mass fraction $f(\delta_{\rm lin} < 0) = 1/2$
so fraction of *overdensites* in collapsed objects of M is

$$F(> \delta_c; M, t) = \frac{f(\delta_{\rm lin} > \delta_c)}{f(\delta_{\rm lin} > 0)} = 2 f(\delta_{\rm lin} > \delta_c) \tag{12}$$

famous factor of two!

Compare mass fraction at M and $M + dM$: difference

$$dF = F(M + dM) - F(M) \simeq \frac{dF}{dM}\, dM \tag{13}$$

$$= \sqrt{\left(\frac{2}{\pi}\right)} \frac{d\sigma(M)^{-1}}{dM} \frac{\delta_c}{\sigma(M)}\, e^{-\delta_c^2/2\sigma^2(M)}\, dM \tag{14}$$

But probability of finding structure M in filter volume $V_{\rm com} = M/\rho_{\rm bg}$ is

$$dF(M) = V \frac{dn}{dM} \, dM = \frac{M}{\rho_{\rm bg}} \frac{dn}{dM} \, dM \qquad (15)$$

and so *PS mass function* is

$$M\frac{dn}{dM} = \frac{\rho_{\rm bg}}{M} \, M\frac{dF}{dM} = \sqrt{\frac{2}{\pi}} \, \frac{d \ln \sigma(M)^{-1}}{d \ln M} \, \frac{\delta_c}{\sigma(M)} \, \frac{\rho_{\rm bg}}{M} \, e^{-\delta_c^2/2\sigma^2(M)}$$

- implicitly also a function of t via $\rho_{\rm bg}(t)$ and $\sigma(M,t)$
- encodes and quantifies hierarchical clustering

from this can immediately find, e.g., distribution of (comoving) density across masses of collapsed objects:

$$\frac{d\rho(M)}{dM} = M \frac{dn}{dM} \qquad (16)$$

Press-Schechter: Summary

Quantitative Output

⋆ Easy to use, very powerful (semi-)analytic mass function

Qualitative Worldview/Limitations

⋆ *every* point lies in exactly one structure:
largest above threshold

⋆ all structures have $\delta_{\rm lin} = \delta_c$: born today!

⋆ PS blind to interior substructure
and formation history of a given object

Q: how to test PS theory?

Q: which structures should be best described? worst?

Tests of Press-Schechter

Versus Numerical Simulations

PS is idealized analytic approximation of hierarchical clustering
assumes true density field δ perfectly mapped onto
linear field $\delta_{\rm lin}$ vis spherical collapse model

Even if underlying CDM, hierarchy idea right, PS approximate
$\rightarrow$ test against numerical simulations w/ non-ideal δ field
results: unreasonably good agreement!

Versus Observations

Best applicable to those just formed: $\sigma(R) \sim \sigma_8 \sim 1$
$\rightarrow$ galaxy clusters! $M \sim 10^{15}\ M_\odot$, and so PS gives

$$n(M) \sim M \frac{dn}{dM} \sim \frac{\rho_0}{M} \nu e^{-\nu^2/2} \sim \frac{\rho_0}{M} \sim 10^{-4}\ \mathrm{Mpc}^{-3} \tag{17}$$

about right! (where $\nu = \delta_c/\sqrt{2}\sigma \sim 1$)
...and works unreasonably well at other scales too

Applications of Press-Schechter

Mergers

PS very powerful because gives mass function vs time:

$$\mathcal{N}(M,t) = M \frac{dn}{dM}(t) \sim \nu(t)\ e^{-\nu^2(t)/2} \tag{18}$$

with

$$\nu(t) = \frac{\delta_c}{\sigma(M,t)} = \frac{\delta_c}{D(t)\sigma_{\rm init}(M)} = \frac{a(t_{\rm init})}{a(t)} \nu_{\rm init} \tag{19}$$

recall: $\sigma_{\rm init}(M)$ *decreases* with M *Q: why?*

So to find time change: just take derivative

$$\dot{\mathcal{N}} \sim |\dot{\nu}|(\nu^2 - 1)e^{-\nu^2/2} \sim \text{creation} - \text{destruction} \tag{20}$$

Q: merging for large, small ν? large, small M?

at fixed time t

$$\dot{\mathcal{N}} \sim |\dot{\nu}|(\nu^2 - 1)e^{-\nu^2/2} \tag{21}$$

small M $\rightarrow$ largest σ: $\nu = \delta_c/\sigma(m) < 1$

$\dot{\mathcal{N}} > 0$: net destruction

and so large M $\rightarrow$ net creation – at expense of small objects

PS Application II: Quasar Abundance

- Quasars must be massive (Eddington limit) black holes at galaxy centers $\rightarrow$ demands $M_{\text{gal}} > M_{\text{bh}} \gtrsim 10^{12} M_\odot$
- Quasars found out to high redshift $z > 3$ (in fact $\gtrsim 7$)

PS: can find number density of objects with $M > 10^{12} M_\odot$ at epoch $z = 3$

$$n_{\text{com}}(> 10^{12} M_\odot; z = 3) = \int_{10^{12} M_\odot} \frac{dn}{dM} dM \sim 10^{-8} \text{ Mpc}^{-3} \tag{22}$$

about right!

Director's Cut Extras

CMB Polarization

Recall: pre-recombination, photons coupled to baryons via Thomson scattering with electrons

Key fact: Thomson scattering is anisotropic and polarized polarization of scattered radiation scales as

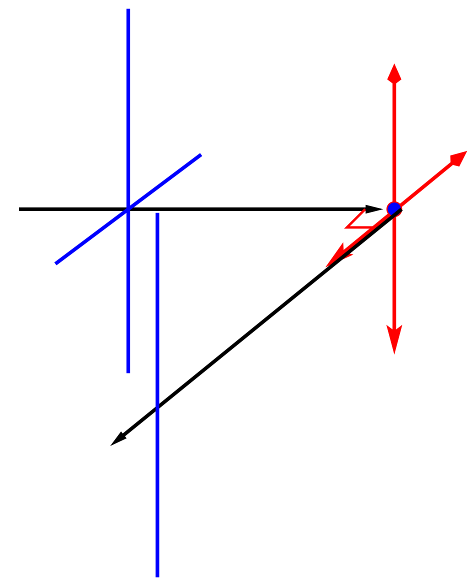
$$\frac{d\sigma_T}{d\Omega} \propto |\hat{\epsilon}_{\rm in} \cdot \hat{\epsilon}_{\rm sc}|^2 = \cos^2\theta \tag{23}$$

where:
$\hat{\epsilon}_{\rm in}$ is *incident photon polarization*
$\hat{\epsilon}_{\rm sc}$ is *scattered photon polarization*
and propagation is transverse: $\hat{\epsilon}_{\rm in} \cdot \hat{n}_{\rm in} = 0$

Q: in what direction is polarization max? min?
Q: why physically? hint: think of e as antenna

max scattered polarization when in plane normal to initial pol'n
zero scattered intensity in direction of initial pol'n

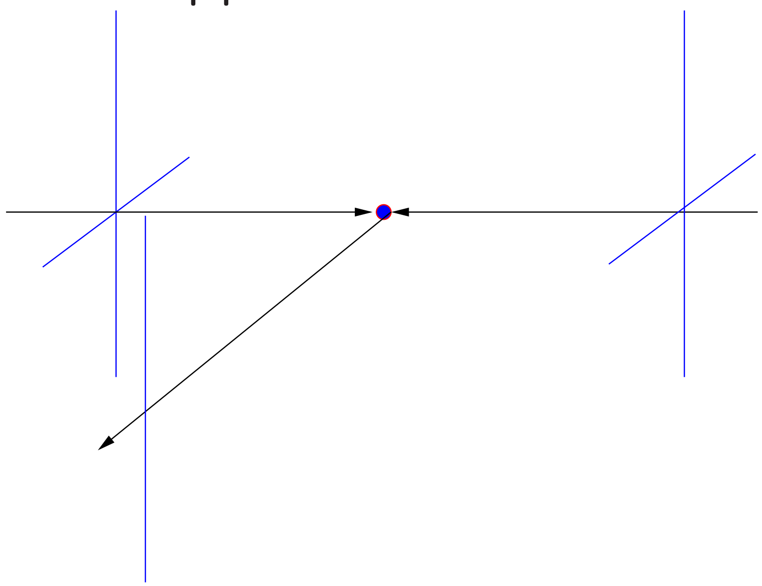
classical picture: e^- as dipole antenna
incident polarized wave accelerates e^-
$\rightarrow$ azimuthally symmetric radiation,
peaks in $\theta = 0$ plane



note: since $\cos^2\theta \propto \cos 2\theta$, scattered rad has 180^0 periodicity
$\rightarrow$ a "pole" every 90^0: quadrupole

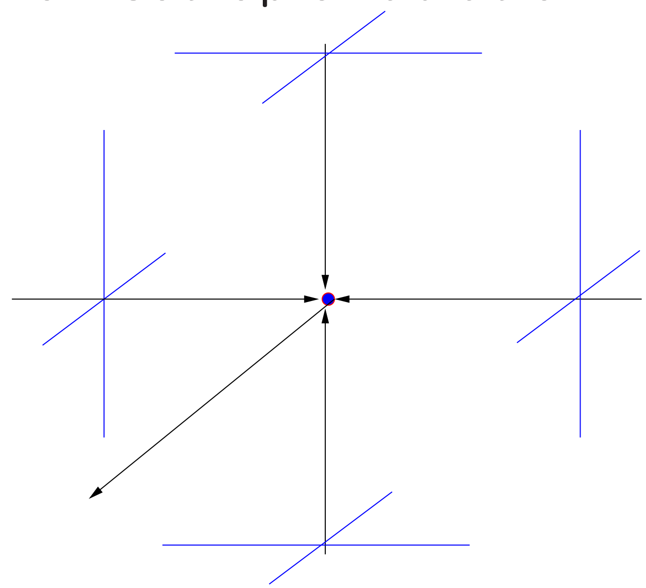
Q: what if unpolarized radiation from 2 opposite directions?
Q: what if isotropic unpolarized radiation?

from opposite incident directions:



still linearly polarized!

for isotropic radiation:



unpolarized!

...as demanded by symmetry

Polarization and Inhomogeneity

Pre-recomb: repeated Thomson scattering
randomizes polarization → CMB unpolarized

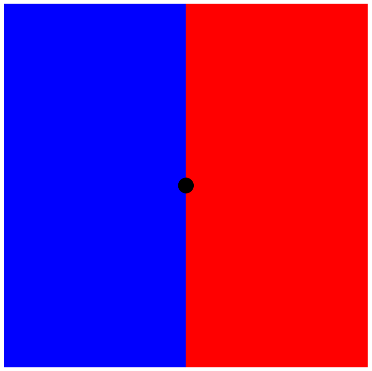
But at recomb, last scattering evens "uncompensated"

- if plasma homogeneous: still no net polarization
- but inhomogeneities → net linear polarization in CMB

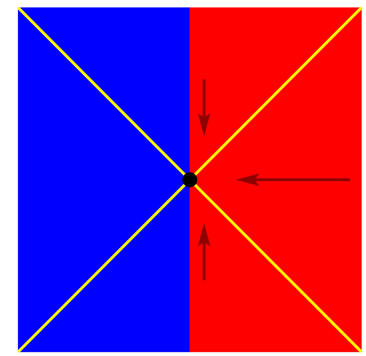
Consider point on hot-cold "wall"

Q: what is scattered polarization? why?

Q: what temperature pattern seen at point?



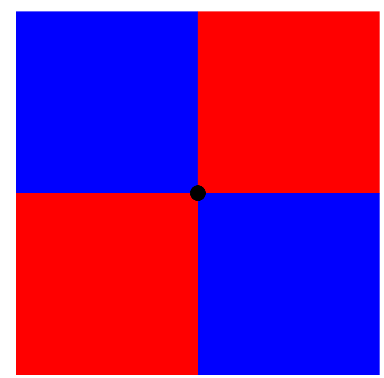
dipole anisotropy:
unpolarized



pattern seen at point:

dipole anisotropy

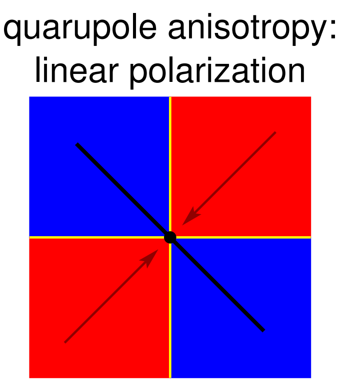
extra polarized radiation from hot region cancels



Now consider point on “ checkerboard vertex”

Q: what is scattered polarization? why?

Q: what temperature pattern seen at point?

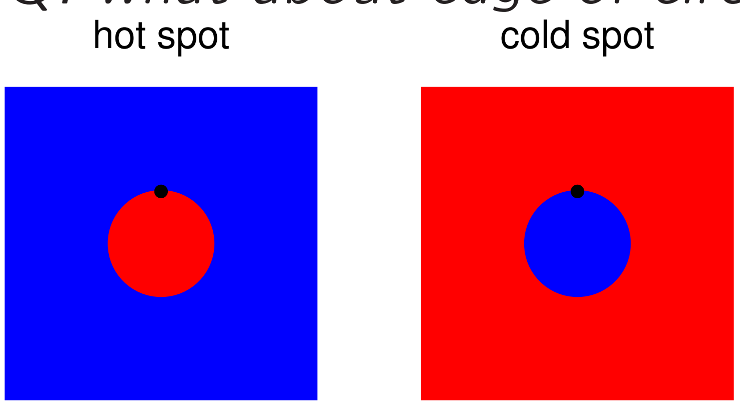


point sees *quadrupole* anisotropy
extra polarization from hot regions
doesn't cancel

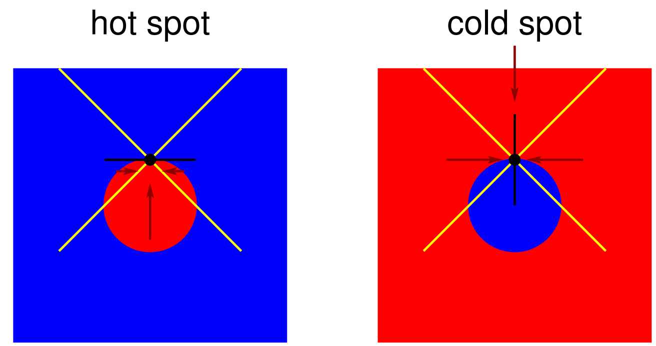
→ net linear polarization towards us, aligned w/ "cold" axis

`www: cool Wayne Hu movie`

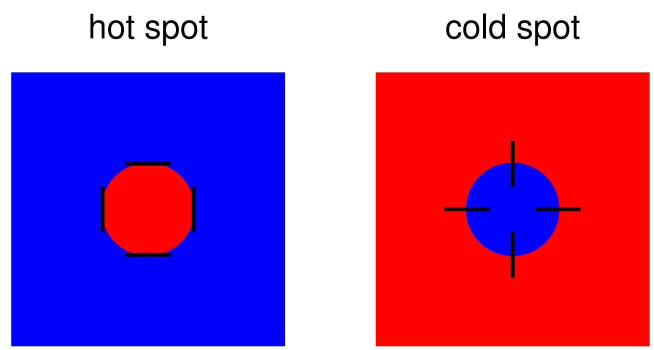
Q: what about edge of circular hot spot? cold spot?



at a single point on edge:



so by symmetry:



polarization tangential (ring) around hot spots
radial (spokes) around cold spots
(superpose to “+” = zero net polarization–check!)

`www: WMAP polarization observations of hot and cold spots`

Note: polarization & T anisotropies *linked*
→ consistency test for CMB theory and hence hot big bang

Polarization: E and B Modes

CMB polarization makes headless vector field on sky
i.e., at each point, polarization vector (possibly zero)
but vector has no "forward/backward" arrow

can decompose polarization field into
- E **modes**: $\mathrm{div}\vec{P} \neq 0$ and $\mathbf{curl}\vec{P} = 0$
- B **modes**: $\mathrm{div}\vec{P} = 0$ and $\mathbf{curl}\vec{P} \neq 0$

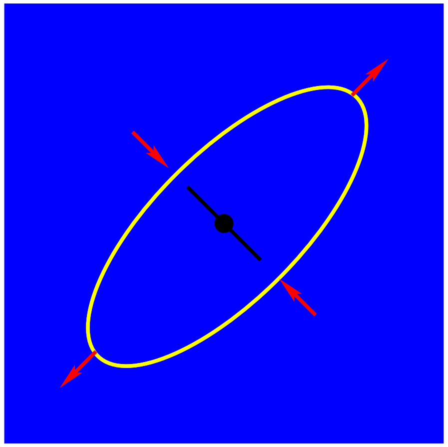
Q: which modes from hot spots? cold spots?

can show:
- *temperature (scalar) perturbations only excite E modes*
- *tensor (gravity wave) perturbations excite both E and B modes*

B Modes and Gravity Waves

recall: gravity waves preserved volume
but stretch and squeeze in $+$ and $\times$ modes

effect on CMB:
velocity perturbation
leads to linear polarization



Cosmology with Clusters: S-Z Effect

clusters contain $T \sim 1/4$ keV gas seen in X-rays
$\rightarrow$ intracluster medium (ICM) fully ionized $\rightarrow$ free e^-
these are targets which scatter photons–including CMB!

Sunyaev & Zel'dovich 1972
consider CMB photon passes thru a cluster
scattering rate per photon $\Gamma_{\rm sc} = n_e \sigma_{\rm T} c$
in time to move increment $ds = c\, dt$, # scatterings is

$$d\tau = \Gamma_{\rm sc} dt = n_e \sigma_{\rm T} ds = \frac{ds}{\lambda_{\rm mfp}} \quad (24)$$

i.e., number of mean free paths $\lambda_{\rm mfp} = (n\sigma)^{-1}$ traversed
total # scatterings: optical depth in line-of-sight thru cluster

$$\tau = \sigma_{\rm T} \int_{\rm los} n_e ds \simeq \sigma_{\rm T} \frac{f_{\rm baryon} M_{\rm cluster}/m_p}{R^2_{\rm cluster}} \sim 0.004 \left(\frac{M_{\rm cluster}}{10^{15} M_\odot}\right) \left(\frac{2\ \rm Mpc}{R_{\rm cluster}}\right)^2$$

Q: which means?

CMB Scattering by Intracluster Gas

mean free path is that for Thompson scattering:
$\ell_\nu^{-1} = \alpha_\nu = n_e \sigma_\mathsf{T}$ independent of frequency
and thus optical depth is integral over cloud sightline

$$\tau_\nu = \int \alpha_\nu \, ds = \sigma_\mathsf{T} \int n_e \, ds \tag{25}$$

thus transmission probability is $e^{-\tau_\nu}$, and so
absorption probability is $1 - e^{-\tau_\nu}$

but for galaxy clusters: $\tau < 10^{-3} \ll 1$,
and so *absorption probability* is just τ
Q: implications?
Q: effect of scattering if electrons cold, scattering is elastic?
Q: what if electrons are hot?

if electrons are hot, they transfer energy to CMB photons
change temperature pattern, in frequency-dependent way

What is net change in energy?
initial photon energy density is $u_0 = u_{\rm cmb} = 4\pi B(T_{\rm cmb})/c$
power transfer per electron is $P_{\rm Compt} = 4(kT_e/m_e c^2)\sigma_{\rm T} c\ u_0$, so

$$\frac{\partial u}{\partial t} = P_{\rm Compt}\ n_e = 4\frac{kT_e}{m_e c^2}\sigma_{\rm T} c\ u_0\ n_e \tag{26}$$

and thus net energy density change

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \tag{27}$$

Q: implications?

CMB energy density change through cluster

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \equiv 4y\ u_0 \qquad (28)$$

- dimensionless **Compton-y parameter**

$$y \equiv \sigma_{\rm T} \int \frac{n_e\ kT_e}{m_e c^2} ds \simeq \tau \frac{kT_e}{m_e c^2} \simeq 3\tau\beta^2 \qquad (29)$$

- note $n_e kT_e = P_e$ electron pressure
  $\rightarrow y$ set by line-of-sight pressure

fractional change in (integrated) energy density $\Delta u/u_0 = 4y$

- positive change $\rightarrow$ (small) net heating of CMB photons
- since $u \propto I$, this also means

$$\frac{\Delta I_{\rm cmb}}{I_{\rm cmb}} = 4y \qquad (30)$$

cluster generated net CMB "hotspot"

Q: expected frequency dependence?

SZ Effect: Frequency Dependence

on average, we expect photons to gain energy
adding intensity at high ν, at the expense of low ν

but note that in isotropic electron population
• some scatterings will reduce energy
• while others will increase it

detailed derivation is involved:
• allow for ordinary and stimulated emission
• include effects of electron energy distribution
• allow for Compton shift in energy
• use Thomson (Klein-Nishina) angular distribution

full equation (Kompaneets and generalization)
describes *"diffusion" in energy (frequency) space*
but key aspect comes from basic Compton property *Q: namely?*

Thermal SZ Effect as a Probe of Galaxy Cluster

in each line of sight
SZ measures Comptonization parameter in a cluster:

$$y = \sigma_{\mathsf{T}} \int \frac{n_e\ kT_e}{m_e c^2} ds = \frac{\sigma_{\mathsf{T}}}{m_e c^2} \int P_e\ ds \approx \frac{\sigma_{\mathsf{T}}\ kT_e}{m_e c^2} \int n_e\ ds \tag{31}$$

direct measurement of *projected pressure* in column
and if T_e known, a measure of electron column density

SZ flux measures

$$\int \cos\theta\ y\ d\Omega \approx \int y\ d\Omega = \frac{\int y\ dA}{D_{\mathsf{A}}^2} \tag{32}$$

where $D_{\mathsf{A}}(z)$ is the (angular diameter) distance

$$\int y\ dA \approx \frac{\sigma_{\mathsf{T}}\ kT_e}{m_e c^2} \int n_e\ ds\ dA \propto M_{\mathsf{gas}} \tag{33}$$

$\rightarrow$ SZ flux gives *intracluster cluster gas mass!* *Q: cosmo apps?*

SZ Effect: Cosmological Applications

- *SZ identifies all clusters without redshift bias!*
  $\rightarrow$ SZ can be used to discover high-z clusters

- SZ + X-ray gives cluster size, gas mass, T_e
  if cluster physics well-understood (Ricker, Vijayaraghavan)
  $\rightarrow$ *cluster mass*

- cluster number density ("abundance") and mass vs z
  i.e., cluster *mass function* a sensitive probe of cosmology

today: clusters are the *largest bound objects*; in early U: rare
number and mass vs time sensitive to *cosmic acceleration*
that competes with *structure growth via gravitational instability*
$\Rightarrow$ clusters probe this competition

Q: so how to find clusters, measure redshifts?

note that SZ redshift independence also means
SZ does not give cluster redshift

Dark Energy Survey key project:
optical images, redshifts of clusters
compare with SZ survey by South Pole Telescope

`www: SPT survey image`

SZ Effect: More Cosmological Applications

even for clusters not clearly imaged in SZ
SZ effect from all clusters still imprinted on CMB
affects $\Delta T_{\rm cmb}$ perturbation pattern on sky

typical angular size of cluster SZ:
for large cluster $\theta_{cluster} \sim R_{\rm cluster}/d_{\rm H} \sim 3\ \mathrm{Mpc}/4\ \mathrm{Gpc} \sim 3\ \mathrm{arcmin}$
i.e., SZ affects small angular scales
in C_ℓ multipole space this corresponds to $\ell \sim 200/\theta_{\rm deg} \sim 4000$

SZ statistical imprint on CMB anisotropies:
exquisitely sensitive measure of *cosmic structure*
for experts: angular power spectrum $C_\ell^{\rm SZ} \propto \sigma_8^7$!

To date: SZ contribution to power spectrum not seen! *Planck*?

Press-Schechter II: Excursion Sets

More sophisticated (and insightful) derivation of same result

Sketch of procedure:

1. given initial density field and (Gaussian) filter window
2. pick a point $\vec{x}$ in space, filter over neighborhood R, mass $M(R)$
3. scan down in mass: at $M \to \infty$, $\sigma(M) \to 0$ *Q: why?*
   and so filtered $\delta(\vec{x})_M = 0$
3. as M decreases, $\sigma(M)$ increases
   filtered $\delta(\vec{x})_M \neq 0$, alternates sign, amplitude
   $\Rightarrow$ $\delta(\vec{x})_M$ is a random walk vs $\sigma(M)$! exactly!
4. can ask: at which M does $\delta(\vec{x})_M$ first cross threshold δ_c
   $\Rightarrow$ this sets M of structure containing point $\vec{x}$
5. repeat for all $\vec{x}$ and average $\to$ PS distribution follows!

Q: limitations/implicit assumptions?

Structure and Horizons

Particle horizons set range for causal physics
including growth of structure
so two requirements for perturbation growth
⋆ perturbation must be inside "horizon," i.e., $\lambda \leq d_H = H^{-1}$
⋆ U. must be matter-dominated: $z < z_{\text{eq}}$

Choreography:
inflation lays down perturbations at z enormous
all frozen in until matter domination , then
• on scales already inside Hubble length at z_{eq}
  δ_m growth stalled until matter-domination
• on superhorizon scales at z_{eq}, δ_m growth begins immediately after $d_H > \lambda$

Today: observe scales in both regimes
Q: What should be the difference?
  What characteristic scale divides these regimes?

Key scale in cosmic structure distribution:
comoving Hubble length at matter-rad equality

$$d_{\rm H,com}(z_{\rm eq}) = \frac{1}{a_{\rm eq} H_{\rm eq}} = \frac{a_{\rm eq}^{1/2} d_{\rm H,0}}{\sqrt{2\Omega_{\rm m}}} \sim 60\ h^{-1}\ {\rm Mpc} \quad (34)$$

corresponding to $k_{\rm eq} = 1/d_{\rm H,com} = 0.02\ h\ {\rm Mpc}^{-1}$
Q: sound familiar?

How do does perturbation growth differ
on scales sub/super horizon at at $z_{\rm eq}$?

in linear regime ($\delta \ll 1$)
linear growth factor: $D(t) = \delta_k(t)/\delta_k(t_{\rm init})$; k-independent

- large scales have linear growth factor $D_0/D_{\rm enter}$
- small scales have grown more in absolute terms but less than linear extrap from horizon entry only grown by $D_0/D_{\rm eq} < D_0/D_{\rm enter}$

Dividing scale at equality horizon:

$\lambda_{\rm eq} = d_{\rm com,hor}(z_{\rm eq}) \sim \eta_{\rm eq}$ and corresponding $k_{\rm eq}$

if smaller scale, horizon entry at pre-eq redshift $z_{\rm enter}$

such that $d_{\rm hor,com}(z_{\rm enter}) = \eta_{\rm enter} = \lambda$

$\rightarrow$ small scales have growth "stunted" by factor

$$\frac{D_{\rm small}}{D_{\rm large}} = \frac{a_{\rm enter}}{a_{\rm eq}} = \left(\frac{\eta_{\rm enter}}{\eta_{\rm eq}}\right)^2 = \left(\frac{\lambda}{\lambda_{\rm eq}}\right)^2 = \left(\frac{k_{\rm eq}}{k}\right)^2 < 1 \tag{35}$$

where we used $D \propto a \propto \eta^2$ in matter-dom

Different scales have not grown by same amount!

$\rightarrow$ to recover initial power spectrum need to account for this

Transfer Function

Theory (initial power spectrum) connected with
Observation (power spectrum processed by growth)
via transfer function–measures "stunting correction"

$$T_k(z) = \frac{\text{present density spectrum}}{\text{extrapolated initial spectrum}} = \frac{\delta_{k,\text{today}}}{D(z)\delta_k(z)} \quad (36)$$

$$\rightarrow \begin{cases} 1 & k < k_{\text{eq}} \\ (k_{\text{eq}}/k)^2 & k > k_{\text{eq}} \end{cases} \quad (37)$$

Note: since $\delta_{k,\text{init}} \sim \delta_{k,0}/T_k$
power spectrum goes as $P_{k,\text{init}} \sim P_{k,0}/T_k^2$

Now apply to observations

Recovering the Initial Power Spectrum

Apply transfer function to invert observed spectrum

Observed power spectrum

- peak at ~ 30 Mpc $\simeq \lambda_{\rm eq}$ (check!)

- for $k < k_{\rm eq}$, $P_{\rm obs}(k) \sim k^1 = P_{\rm init}(k)$
  $\rightarrow$ scale invariant! (check!)

- for $k > k_{\rm eq}$, turnover in power spectrum (check!)
  quantitatively: $P_{\rm obs}(k) \rightarrow k^{-3}$
  so $P_{\rm init} \sim P_{\rm obs}/T^2 \sim k^4 P_{\rm obs} \sim k$
  also scale invariant (check!)

observed power spectrum consistent with
gravitational growth of scale-invariant spectrum!