Astro 507

Lecture 6

Feb. 3, 2020

Announcements:

- **Problem Set 1 due this Friday, Feb. 7**
  Problem 2 involves gathering data online from SDSS
- Office Hours: Instructor 3–4 pm Wed, or by appointment
  TA: noon–1pm Thursday, or by appointment

Last time: Cosmodynamics I–Newtonian Cosmology

result: the right answer–Dr. Friedmann's famous equation

Suitable for framing, tweets, T-shirts, tattoos...

Q: what are the Friedmann eqs? why are they useful?

Q: Friedmann for an empty universe with $\kappa = 0$*? with* $\kappa > 0$*?*

An Empty Universe

for now, define empty: no matter, radiation, fields, etc

$\rho = 0$ and $P = 0$, giving Friedmann eqs

$$\left(\frac{\dot{a}}{a}\right) = -\frac{\kappa c^2}{R^2} \quad (1)$$

$$\frac{\ddot{a}}{a} = 0 \quad (2)$$

★ $\ddot{a} = 0$: *an unaccelerated, universe!*

$(\dot{a}/a)^2 > 0$ requires $\kappa = 0$ or 1

- $\kappa = 0$: $\dot{a} = 0 \rightarrow a = 1$ always

  a static universe! not ours!

★ $\kappa = -1$: $\dot{a}^2 = c^2/R^2$, solve:

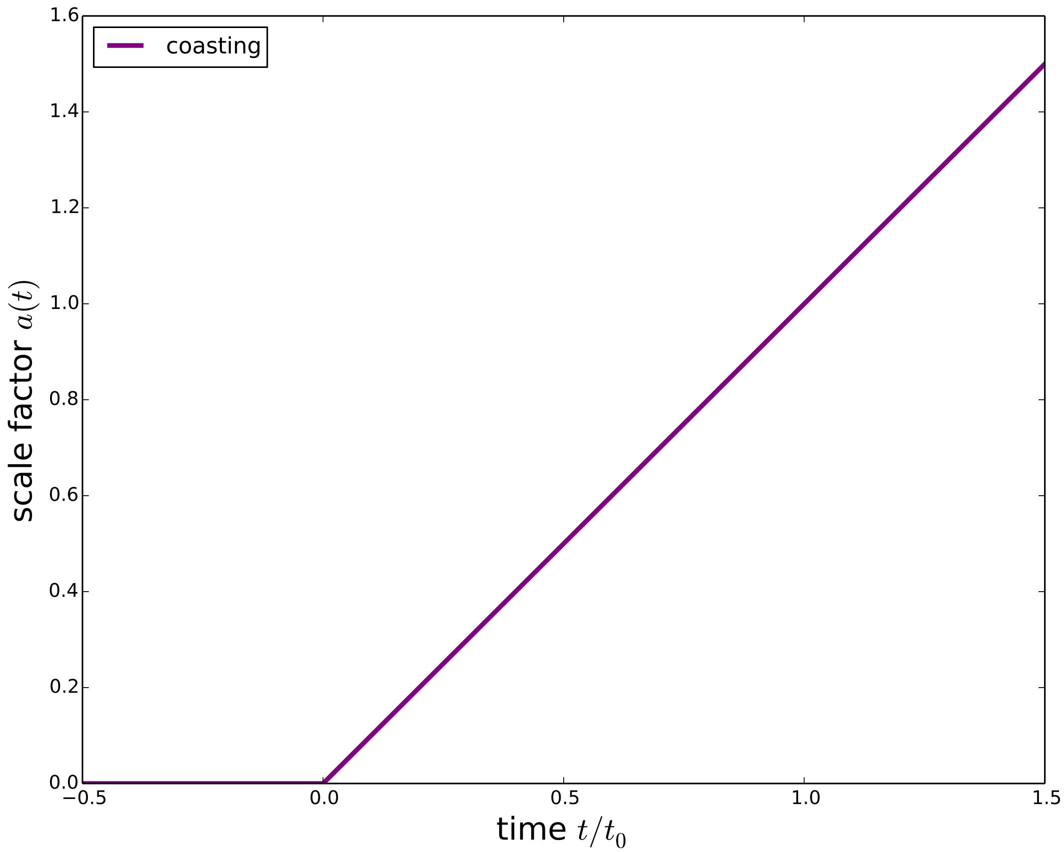
$$a(t) = \frac{ct}{R}$$

Q: assumptions in solution? physical interpretation? fate? $H(t)$?

empty $\kappa = -1$ universe: dull, but useful for reference

$$a(t) = \frac{ct}{R}$$

- solution assumed expansion, not contraction
  and put $a = 0$ at $t = 0$: big bang! more later on this
- $a \propto t$ linear growth time
  test particle separation grows with constant velocity!
  unaccelerated universe $\rightarrow$ coasting motion!
  this is essentially the Milne universe!
- fate/destiny = behavior at large t
  here $a(t)$ grows without bound: expand forever
- Hubble rate: $H = \dot{a}/a = 1/t$ as in Milne, not constant
  evaluate today: $H_0 = 1/t_0 \rightarrow t_0 = t_{\mathrm{H},0}$, as in Milne
- today $a = 1 = ct_0/R$: curvature scale $R = ct_0 = d_{\mathrm{H}}$



Q: now how do we add matter?

A Matter-Only Universe

consider a universe containing *only* non-relativistic matter
Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2}\frac{1}{a^2} \qquad (3)$$

$$= \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{\kappa c^2}{R^2} a^{-2} \qquad (4)$$

For $\kappa = 0$: "Einstein-de Sitter"

$$(\dot{a}/a)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} \qquad (5)$$

evaluate today: $H_0^2 = 8\pi G \rho_0 / 3$

$$a^{1/2} da = H_0 \, dt \qquad (6)$$

$$2/3 \; a^{3/2} = H_0 \, t \qquad (7)$$

Q: implicit assumptions in solution?
Q: fate/destiny/far future of an Einstein-de Sitter universe?

Einstein-de Sitter:

$$t = \frac{2}{3} a^{3/2} H_0^{-1} \qquad (8)$$

$$a = \left(\frac{3}{2} H_0 t\right)^{2/3} = \left(\frac{t}{t_0}\right)^{2/3} \qquad (9)$$

Now unpack the physics:

- boundary condition: $a = 0$ at $t = 0 \rightarrow$ "big bang"
- $a \propto t^{2/3}$ *Q: interpretation?*
- evaluate Hubble parameter

$$H = \frac{\dot{a}}{a} = \frac{2}{3}\frac{1}{t} \qquad (10)$$

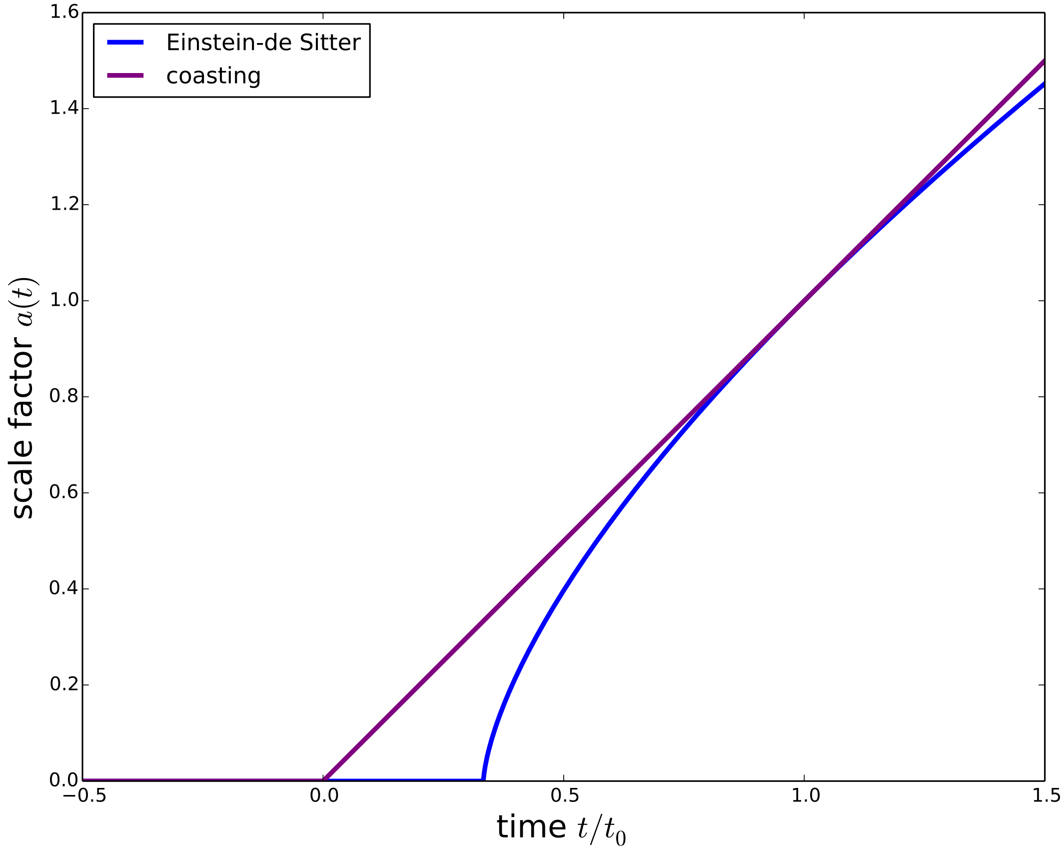
Q: interpretation?

- present age:

$$t_0 = \frac{2}{3}\, H_0^{-1} = \frac{2}{3}\, t_H \qquad (11)$$

Hubble time t_H sets scale

Q: note that $t_0 < t_H$: why?



⇝ require both have $a(t_0) = 1$ and same $H_0 = \dot{a}(t_0)$

A Matter-Only Universe: Physical Interpretation

Einstein-de Sitter evolution:

- $a \propto t^{2/3}$: universe expands without limit
- fate: $a \to \infty$ as $t \to \infty$: universe expands forever!
- expansion rate $H = 2/3t$: slows, $H \to 0$ as $t \to 0$
  reflects $\ddot{a} < 0$: universe decelerates
  due to gravitational attraction of matter

Other Einstein-de Sitter fun facts:

- U. half its present age at $a = 2^{-2/3} = 0.63$
- objects half present separation (and $8\times$ more compressed)
  at $t = 2^{-3/2} t_0 = 0.35 t_0$
- using measured value of H_0, calculate $t_0 = 8.9$ Gyr
  but know globular clusters have ages $t_{gc} \gtrsim 12$ Gyr *Q: huh?*

Matter and Curvature

What if $\kappa = +1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{c^2}{R^2}a^{-2}$$

a cannot grow without bound *Q: why?*

Q: what is $a_{\rm max}$?

Q: evolution after $a = a_{\rm max}$? cosmic fate?

What if $\kappa = -1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} + \frac{c^2}{R^2}a^{-2}$$

Q: what is $a_{\rm max}$? cosmic fate?

Matter and Curvature

if $\kappa = +1$: *positive curvature*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{c^2}{R^2} a^{-2}$$

- $H = \dot{a}/a = 0$ when $a = a_{\text{max}} = 8\pi G R^2 / 3c^2$
- but for all t, all a: $\ddot{a}/a = -4\pi G\rho/3 < 0$
  $\rightarrow$ after maximum, $H < 0 \rightarrow$ *universe contracts*

fate: collapse continues back to $a = 0$: **"big crunch!"**

if $\kappa = -1$: negative curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} + \frac{c^2}{R^2} a^{-2}$$

$H > 0$ for all $a \rightarrow a$ grows without bound

fate: expand forever–**"big chill"!**

at large t, "curvature-dominated": $a(t) \rightarrow ct/R$

Q: how can we tell what our κ value is?

Geometry, Density, and Dynamics

rewrite Friedmann

$$1 = \frac{8\pi G\rho}{3H^2} - \frac{\kappa c^2}{R^2}(aH)^{-2} = \Omega - \frac{\kappa c^2}{R^2}(aH)^{-2} \tag{12}$$

where the density parameter is

$$\boxed{\Omega = \frac{\rho}{\rho_{\rm crit}}} \tag{13}$$

where the critical density is

$$\boxed{\rho_{\rm crit} = \frac{3H^2}{8\pi G}} \tag{14}$$

Note: for a particular density component ρ_i
corresponding density parameter is $\Omega_i = \rho_i/\rho_{\rm crit}$
total $\Omega \equiv \Omega_{\rm tot}$ sums all species: $\Omega = \sum_i \Omega_i$

Note that

$$\kappa = \left(\frac{aHR}{c}\right)^2 (\Omega - 1) = (\text{pos def}) \times (\Omega - 1)$$

geometry (and fate*) of Universe $\Leftrightarrow \kappa \Leftrightarrow \Omega - 1$

if $\Omega = 1$ ever:

- $\Omega = 1$ always; $\kappa = 0 \rightarrow$ no curvature, expand forever

if $\Omega < 1$ ever:

- $\Omega < 1$ always; $\kappa = -1 \rightarrow$ negative curvature, expand forever

if $\Omega > 1$ ever:

- $\Omega > 1$ always; $\kappa = +1 \rightarrow$ positive curvature, recollapse

Q: but if Ω just a stand-in for κ, why useful?

*κ always gives geometry, but κ and fate decoupled if $\Lambda \neq 0$

Geometry and Fate are Knowable!

we saw: κ found from Ω

and: we can determine $\Omega \propto \rho/H^2$
from *locally measurable quantities* ρ and H:
$\rightarrow$ cosmic fate & geometry knowable!
...and become *experimental questions!*

But recall:
so far, only have considered non-relativistic matter
definitely an incomplete picture
$\rightarrow$ at minimum, must include photons!

To Be or Not to Be Relativistic

for a particle ("species") of mass m
relativistic status set by comparison: typical speed v vs c
equivalent to comparing: typical $E_{\rm kin}$ vs mc^2
but if thermal, $E_{\rm kin} \sim kT$
$\rightarrow$ relativistic: $kT \gg mc^2$ $\rightarrow$ non-relativistic: $kT \ll mc^2$

massless particles

if $m = 0$: always have $v = c$ $\rightarrow$ forever relativistic

massive particles

if $m > 0$: *always* a time in Early U when $kT \gg mc^2$
$\rightarrow$ massive particles born relativistic, become non-rel!
$\rightarrow$ relativistic status is time-dependent!

Q: are there species which are always *relativistic? non?*
Q: what is relativistic, non-rel today?

Today: $kT_{\mathrm{CMB},0} \sim 10^{-4}$ eV

always: photons relativistic because $m_\gamma = 0$

gravitons also massless (if they exist)

clearly: $m_e c^2, m_p c^2 \gg kT_0 \rightarrow$ non-relativistic today!

but *were* relativistic in early U

but what about *neutrinos?*

we know: 3 massive species exist

do not (yet!) know mass of any species

but we *do* know their mass differences

for experts: oscillation experiments measure $\delta m_{ij}^2 = m_i^2 - m_j^2$

which set a laboratory-based *lower limit*:

heaviest neutrino must have $m_\nu > 0.04$ eV

$\rightarrow$ *at least one ν species non-relativistic today!*

$\rightarrow$ contributes to Ω_{matter}

Redshifts I

quick-n-dirty: wavelengths are lengths! ..it's right there in the name!
$\rightarrow$ expansion stretches photon λ

$$\lambda \propto a$$

if *emit* photon at $t_{\rm em}$, then at later times

$$\lambda(t) = \lambda_{\rm emit} \frac{a(t)}{a(t_{\rm em})} \tag{15}$$

if *observe* later, $\lambda_{\rm obs} = \lambda_{\rm em}\ a_{\rm obs}/a_{\rm em}$
measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \tag{16}$$

high z $\leftrightarrow$ small a $\leftrightarrow$ distant past

Newtonian Derivation of Redshift: Hubble & Doppler

slower-n-cleaner: non-relativistic Doppler
non-rel Doppler sez:

$$\frac{\delta\lambda}{\lambda} \equiv z = \frac{v}{c} \tag{17}$$

Hubble sez:

$$cz = Hr \tag{18}$$

Together

$$\frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{19}$$

But light travels distance r in time $\delta t = r/c$, so

$$\frac{\delta\lambda}{\lambda} = H\delta t = \frac{\dot{a}\delta t}{a} = \frac{\delta a}{a} \tag{20}$$

for arriving light, fractional λ change = fractional a change!

Scale Factor and Redshift

$$a = \frac{1}{1+z}$$
$$z = \frac{1}{a} - 1$$

recordholders to date–most distant objects `www: recordholders`

- farthest quasar: $z = 7.085$
- farthest gamma-ray burst: $z \approx 9.2$
- farthest galaxy: $z \sim 12$ (photometric data only)

For $z = 12$, *when light emitted:*
$\rightarrow$ scale factor was $a = 0.08$
interparticle (intergalactic) distances 8% of today!
$\rightarrow$ galaxies were 13 times closer
squeezed into volumes 2200 times smaller!
$\rightarrow$ age: $t = 2/3\ \Omega_{\rm m}^{-1/2} t_{\rm H}/(1+z)^{3/2} = 0.026\ t_{\rm H} = 370$ Myr

Q: implications of seeing galaxies and GRBs at such z?

Redshifts and Photon Energies

in photon picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \tag{21}$$

$\to \gamma$ energy redshifts

Consequences:

▷ *Q: photon energy density $\varepsilon(a)$?*

▷ if thermal radiation,

Q: $T \leftrightarrow \lambda$ connection?

Q: expansion effect on T?

Relativistic Species

Photon energy density: $\varepsilon_\gamma = E_\gamma \, n_\gamma$
average photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow$ $\boxed{\varepsilon_\gamma \propto a^{-4}}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\rm max}$
but since $\lambda \propto a \rightarrow$ then $\boxed{T \propto 1/a}$

Consequences:
- $\varepsilon_\gamma \propto T^4$: Boltzmann/Planck!
- T decreases $\rightarrow$ U cools!
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)