Astro 507

Lecture 7

Feb. 5, 2020

Announcements:

- **Problem Set 1 due this Friday, Feb. 7**
  Problem 2 involves gathering data online from SDSS
  Q2 hint: careful with units!
  it may be useful to convert arcmin^{-2} $\rightarrow$ rad$^{-2} = sr^{-1}$

- Office Hours: Instructor 3–4 pm Wed, or by appointment
  TA: noon–1pm Thursday, or by appointment

Cosmological Physics Colloquium 4pm today:
Renee Hlozek, U Toronto
"Constraining Axion Physics With Small-Scale CMB Measurements"
particle physics + dark matter + CMB observations!

Last Time

Last time: Friedmann and the contents of the universe

▷ ρ_{crit} *Q: definition? interpretation? how measure?*

▷ Ω and Ω_i *Q: definition? interpretation? how measure?*

▷ radiation: what counts? examples?
λ response to expansion?
a vs z relation?

Newtonian Derivation of Redshift: Hubble & Doppler

slower-n-cleaner: non-relativistic Doppler
non-rel Doppler sez:

$$\frac{\delta\lambda}{\lambda} \equiv z = \frac{v}{c} \tag{1}$$

Hubble sez:

$$cz = Hr \tag{2}$$

Together

$$\frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{3}$$

But light travels distance r in time $\delta t = r/c$, so

$$\frac{\delta\lambda}{\lambda} = H\delta t = \frac{\dot{a}\delta t}{a} = \frac{\delta a}{a} \tag{4}$$

for arriving light, fractional λ change = fractional a change!

Scale Factor and Redshift

$$\begin{aligned} a &= \frac{1}{1+z} \\ z &= \frac{1}{a} - 1 \end{aligned}$$

recordholders to date–most distant objects `www: recordholders`

- farthest quasar: $z = 7.085$
- farthest gamma-ray burst: $z \approx 9.4$ (photometric data only)
- farthest galaxy: $z \sim 12$ (photometric data only)

For $z = 12$, *when light emitted:*
$\rightarrow$ scale factor was $a = 0.08$
interparticle (intergalactic) distances 8% of today!
$\rightarrow$ galaxies were 13 times closer
squeezed into volumes 2200 times smaller!
$\rightarrow$ age: $t = 2/3\ \Omega_{\rm m}^{-1/2} t_{\rm H}/(1+z)^{3/2} = 0.026\ t_{\rm H} = 370\ {\rm Myr}$

Q: implications of seeing galaxies and GRBs at such z?

Redshifts and Photon Energies

in photon picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \tag{5}$$

$\rightarrow$ a photon's energy decreases with cosmic expansion

Consequences:

▷ *Q: photon energy density $\varepsilon(a)$?*

▷ if thermal radiation,

Q: $T \leftrightarrow \lambda$ connection?

Q: expansion effect on T?

Relativistic Species

Photon energy density: $\varepsilon_\gamma = E_\gamma \; n_\gamma$
average photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow$ $\varepsilon_\gamma \propto a^{-4}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\rm max}$
but since $\lambda \propto a$ $\rightarrow$ then $T \propto 1/a$

Consequences:

- $\varepsilon_\gamma \propto T^4$: Boltzmann/Planck!
- T decreases $\rightarrow$ U cools!
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)

Radiation and Friedmann

definition: to cosmologist, **radiation** $\equiv$ *relativistic* matter
photons or *any* particle with $v \sim c$, $E \sim T \gg mc^2$
energy density $\varepsilon_{\rm rad} \propto a^{-4}$

gravitational effects: due to *equivalent mass density*

$$\varepsilon = \rho c^2$$

so Friedmann radiation term: $\rho_{\rm rad} \propto a^{-4}$

Add radiation to Friedmann energy equation:

$$\rho = \rho_{\rm total} = \rho_{\rm m} + \rho_{\rm rad} = \rho_0(\Omega_{\rm m,0} a^{-3} + \Omega_{\rm r,0} a^{-4})$$

note: today, $\Omega_{\rm r,0} = 4.15 \times 10^{-5} h^{-2} \ll 1$

Q: radiation pressure–relation to energy density?

Radiation Pressure

Also: Maxwell (classical E&M) says pressure

$$P_{\text{EM}} = \frac{1}{3}\varepsilon_{\text{EM}} \tag{6}$$

Max Planck agrees! (photon picture gives same answer)

⋆ include this in Friedmann acceleration

⋆ put $V = a^3$, then $\varepsilon \propto V^{-4/3}$, and

$$d(\varepsilon_{\text{rad}} V) = -\frac{1}{3}\,\varepsilon_{\text{rad}}\,dV = -P_{\text{rad}}\,dV$$

Q: physical interpretation?

∞

1st Law and Equation of State

Generalize: Cosmological "**1st Law of Thermodynamics**"

$$d(\rho c^2 a^3) = -P\ d(a^3) \tag{7}$$

GR verifies this is correct!

$\Rightarrow$ reconciles Friedmann energy, accel eqs:

ensures that $\ddot{a} = d\dot{a}/dt$ (try it!)

to solve, need to relate P to ρc^2 $\rightarrow$ **equation of state**

- non-rel matter: $P_{\rm m} \ll \rho_{\rm m} c^2 \approx 0$ *Q: why? e.g., ideal gas?*
- radiation: $P_{\rm rad} = \rho_{\rm rad} c^2/3$

Q: useful generalization?

Equation of State Parameter

Cosmological 1st Law has a ρc^2 term and a P term
for each cosmic species

so for each species, useful to define
"*equation of state parameter*" w such that

$$P = w\rho c^2$$

Q: $w_{\rm matter}$*?,* $w_{\rm rad}$*?*

Can solve 1st Law eq for matter with **constant** w:

$$\rho_w \propto a^{-3(1+w)} \tag{8}$$

Q: what if $w = 0, +1/3, -1$*?*

Cosmological Constant

Einstein (1917) "**cosmological constant**" Λ
a new constant of nature

acts as substance with $w = -1$

- $P_\Lambda = -\rho_\Lambda c^2 < 0$!?
  negative pressure !?!

- $\rho_\Lambda \propto a^0 = const$
  constant energy density (and pressure) !?!
  i.e., expansion does not change ρ_Λ, P_Λ!
  "vacuum energy"

Ingredients of a Minimally Realistic(?) Universe

For sure, the universe contains:

- *Matter Q: evidence?*
  $\rho_{\mathrm{m}} \propto a^{-3}$
- Radiation *Q: evidence?*
  $\rho_{\mathrm{r}} \propto a^{-4}$

Quite possibly, the universe could contain:

- Curvature
  curvature term $\propto a^{-2}$
- Cosmo Const (or worse!)
  $\rho_{\Lambda} \propto a^0 = const$

So: "minimal" but also "realistic" account of U
must include these pieces: $\rho = \rho_{\mathrm{tot}} = \sum_i \rho_i$

Cosmodynamics in a Minimally Realistic(?) Universe

for a "minimally realistic" universe, Friedmann sez:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{r,0} a^{-4} + \rho_{m,0} a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2} a^{-2}$$

$$= H_0^2 \left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda + (1 - \Omega_{tot}) a^{-2}\right]$$

Q: limiting cases?

Limiting cases: one term $\gg$ all others (PS1)
component i dominates when

$$\rho_{\text{tot}} \approx \rho_i \gg \rho_{\text{other}} \tag{9}$$

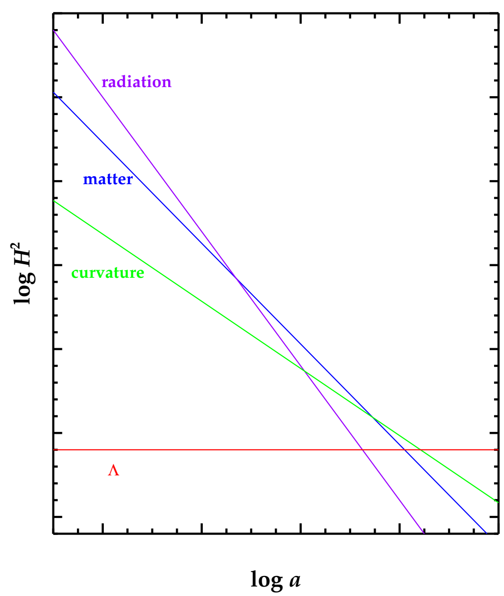
- radiation-dominated: $a_{\text{rd}} \sim t^{1/2}$
- matter-dominated: $a_{\text{md}} \sim t^{2/3}$
- curvature-dominated $\kappa = -1$; *Q: why?* $a_{\text{cd}} \propto t^1$
- Λ-dominated: $a_{\lambda d} \propto e^{+H_\Lambda t}$

Q: which component most important at early times? late times?

The Cosmic Past

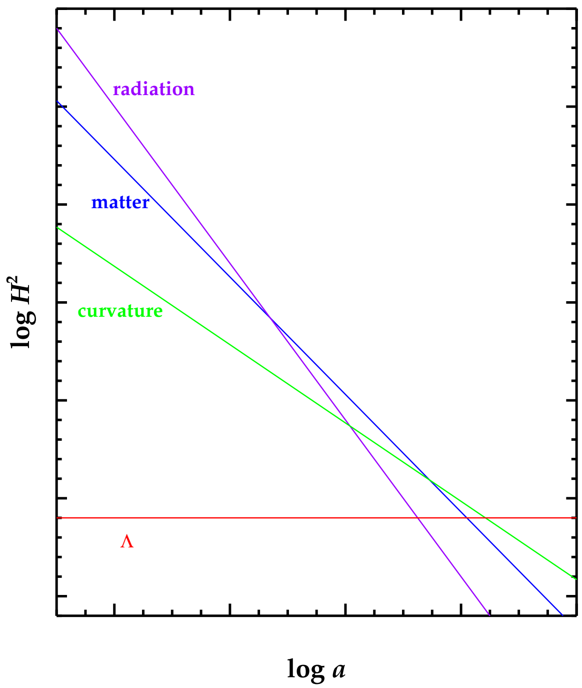
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_\Lambda) - \frac{\kappa c^2}{R^2}a^{-2}$$

$$= \frac{8\pi G}{3}\left(\rho_{r,0}a^{-4} + \rho_{m,0}a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2}a^{-2}$$



Curves for specific choices of parameters $(\rho_{m,0}, \rho_{r,0}, R, \Lambda)$

Q: change if these have different values?



Mix-n-match:

Q: evolution if only matter & rad? Ω?

Q: ... if matter, rad, and curv(±)? Ω?

Q: ... if matter, rad, and Λ? Ω?

Q: ... if matter, rad, curv, and Λ? Ω?

Menu at Al Friedmann's Cosmo Café

Possible Histories of the Universe

Matter + Radiation only: ($\Omega = 1$)

rad-dom $\rightarrow$ matter-dom; expand forever

Matter + Radiation + Curvature($-$): ($\Omega < 1$)

RD $\rightarrow$ MD $\rightarrow$ CD; expand forever

Matter + Radiation + Curvature($+$): ($\Omega > 1$)

RD $\rightarrow$ MD $\rightarrow$ CD $\rightarrow$ reverse; recollapse

Matter + Radiation + Λ: ($\Omega = 1$)

RD $\rightarrow$ MD $\rightarrow$ ΛD: expand forever *exponentially!*

Matter + Radiation + Λ + curv: ($\Omega \neq 1$)

many possibilities! fate depends on detailed composition

Radiation and the Early Universe

note: radiation *always wins out* at early times
$\Rightarrow$ Early U is radiation-dominated
Q: why?

later evolution (which components dominate)
depends on cosmic ingredients
and their relative amounts

Density and Destiny

Enough generalities! What about *our* real Unvierse?
Fate (and geometry) of U. depend on
current values of $\Omega_{i,0} = \rho_{i,0}/\rho_{\rm crit,0}$
and $\Omega_0 = \sum \Omega_i$ where

$$\begin{aligned}
\rho_{\rm crit,0} &= \frac{3H_0^2}{8\pi G} \\
&= 1.9 \times 10^{-29}\ h^2\ {\rm g/cm^{-3}} \approx 10^{-29}\ {\rm g/cm^{-3}} \\
&= 2.78 \times 10^{11}\ h^2\ M_\odot\ {\rm Mpc^{-3}} \approx 1.4 \times 10^{11}\ M_\odot\ {\rm Mpc^{-3}} \\
&\approx 6\ {\rm H\,atoms\ m^{-3}}
\end{aligned}$$

Empirical question:

- is $\rho_{\rm tot,0}$ bigger or smaller than this number?
- *density is destiny! weight is fate!*

Cosmic Geometry and Evolution

Consider a universe with $\Omega \neq 1$

Friedmann says

$$\Omega(t) - 1 = \frac{\kappa c^2}{R^2 a^2 H^2} = \frac{\kappa c^2}{R^2 \dot{a}^2} \propto \frac{1}{\dot{a}^2} \tag{10}$$

i.e., Ω *changes* with time

Q: is $|\Omega - 1|$ *increasing or decreasing?*
Q: limiting values of Ω *at large* t*?*
Q: physical interpretation of these limits?
Q: timescale for Ω *to change?*
Q: implications for Ω_0*?*

The Evolution of Ω

Time change of $|\Omega - 1| \propto 1/\dot{a}^2$ is

$$\frac{1}{|\Omega - 1|} \frac{d}{dt} |\Omega - 1| = \dot{a}^2 \frac{d}{dt} \frac{1}{\dot{a}^2} \tag{11}$$

$$= -2 \frac{\ddot{a}/a}{H^2} H = 2 \, q \, H \tag{12}$$

where *acceleration parameter* $q = -(\ddot{a}/a)/H^2$
Q: why sign choice in q definition?

- generally, $|q| \sim 0.1 - 10$, so
  $|\Omega - 1|$ changes on timescale $1/2|q|H \sim 1/H = t_H \sim t$

- if $\ddot{a} < 0$: ordinary attractive gravity, *decelerating* U
  then $|\Omega - 1|$ *increasing* with time
  $\rightarrow \Omega$ driven increasingly away from 1
  Q: unless...?

What is Ω_0?

Procedure 0: Pure Theory

$\Omega = \rho/\rho_{\text{crit}} \sim \rho(t)/H^2(t)$ evolves

- if ever $\Omega = 1$, stays 1 always
- otherwise: $\Omega \to 0$ or ∞
- physically: expand forever or recollapse
  occurs on cosmic timescale t: current age

$\Omega = 1$ is the only stable value

do the experiment: look around room

$\Omega \neq 0, \infty \rightarrow$ $\Omega = 1$!

else conspiracy: we live just when $\Omega \sim 1$

"Dicke coincidence"

What is Ω_0?

Procedure I: Galaxy Surveys

Goal: measure $\rho_0 \rightarrow$ infer Ω_0

Q: What is $\Omega_{\text{this room}}$?

Q: Why can't we use $\rho_{\text{this room}}$?

Q: What is needed?

Q: What do galaxy surveys actually measure?

Q: How can we bridge the gap?

Cosmic Density Measurement Procedure I:
Mass-to-Light Ratios

Seems simple...
1. find fair sample of U., with some volume V
2. if measure total mass M, $\rightarrow \rho = M/V$

...but telescopes don't measure mass, rather: *luminosity* L
1. find cosmic **luminosity density** $\mathcal{L} = L/V$
2. then find cosmic ratio of mass to luminosity:
   mass-to-light ratio $M/L \equiv \Upsilon$
3. solve for mass density $\rho = \Upsilon \; \mathcal{L}$

Galaxy surveys: $\mathcal{L}_{\rm obs} \sim 2 \times 10^8 \; h \; L_\odot \; {\rm Mpc}^{-3}$
...which you will ~verify in PS1!

Need "fair sample" of mass-to-light ratio Υ

Q: how to measure this?

cosmic mass/light sample: galaxies including dark halos

flat rotation curves $v(r) \sim const$

`www: rotation curve`

Newtonian gravity, dynamics apply:
circular motion: $v^2/r \sim g \sim GM_{\text{enclosed}}(r)/r^2$
Q: expected behavior for $r >$ visible matter?

Instead: find $v \approx const$ well beyond visible matter
"flat rotation curves"
$\Rightarrow M(r) \sim v^2 r/G \sim r$ for $r \gg r_{\text{vis}}$!
dark halo! typically $M_{\text{halo}} \sim 5 - 10 M_{\text{vis}}$
summing observed light, total dynamical mass:

$$\Upsilon_{\text{halo}} \lesssim 25 h M_\odot / L_\odot \rightarrow \Omega_{\text{halo}} \lesssim 0.02 \ll 1$$

Q: implications? what if this is a fair sample?
Q: why would/wouldn't it be?

cosmic mass/light sample: galaxy clusters

can find cluster $M_{\rm tot}$ from several methods

e.g., `www: cluster gravitational lens`

$\Upsilon_{\rm cluster} \sim 300h \rightarrow \Omega_{\rm cluster} \sim 0.25h^{-1} \sim 0.3$

Note: since $\Upsilon_{\rm cluster} > \Upsilon_{\rm halos}$

$\rightarrow$ immediately conclude that *halos are not fair sample*

$\rightarrow$ i.e., halos miss extra dark matter on larger scales

$\rightarrow$ hints for galaxy formation...

...but clusters have $\delta\rho/\rho_0 \sim 1$

$\rightarrow$ largest bound objects

$\rightarrow$ should be fair sample:

$\Rightarrow \Omega_{\rm matter} \sim 0.3$ (including DM!)

Cosmic Density Measurement Procedure II:

Microwave background anisotropies

CMB temperature anisotropies sensitive to cosmic geometry

`www: Planck 2013 results + other observations (BAO)`

$$\Omega_\kappa \equiv 1 - \Omega_0 = 0.0005 \pm 0.0033$$

$$\boxed{\Omega_0 = 1.0005 \pm 0.0033!}$$

$\Rightarrow \Omega_0 = 1$ to $\sim 0.3\%$ level!!!

$\Rightarrow$ *a flat universe! theory prejudice correct!*

but: $\Omega_{\rm matter} \approx 0.27$ (including DM!)

$\rightarrow \Omega_{\rm other} = 0.73$?!?

Who ordered that? What is the other, dominant component?

Λ? "dark energy" ?!?