Astro 507
Lecture 8
Feb. 7, 2020

Announcements:

- **Problem Set 1 due 5pm today**
- **Preflight 2 posted, due noon next Friday**

Last time

- radiation in the Universe
  energy density ϵ va a? ρ?
  if thermal: $T(a)$? ρ?
- pressure: *Q: cosmic first law of thermodynamics?*
  Q: cosmic equation of state?
- minimal ingredients for a realistic cosmology?

Ingredients of a Minimally Realistic(?) Universe

For sure, the universe contains:

- *Matter Q: evidence?*
  $\rho_{\mathrm{m}} \propto a^{-3}$
- Radiation *Q: evidence?*
  $\rho_{\mathrm{r}} \propto a^{-4}$

Quite possibly, the universe could contain:

- Curvature
  curvature term $\propto a^{-2}$
- Cosmo Const (or worse!)
  $\rho_{\Lambda} \propto a^{0} = const$

So: “minimal” but also “realistic” account of U
must include these pieces: $\rho = \rho_{\mathrm{tot}} = \sum_i \rho_i$

Cosmodynamics in a Minimally Realistic(?) Universe

for a “minimally realistic” universe, Friedmann sez:

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\left(\rho_{\mathrm{r},0}a^{-4} + \rho_{\mathrm{m},0}a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2}a^{-2} \\ &= H_0^2\left[\Omega_\mathrm{r}a^{-4} + \Omega_\mathrm{m}a^{-3} + \Omega_\Lambda + (1-\Omega_\mathrm{tot})a^{-2}\right] \end{aligned}$$

Q: limiting cases?

Limiting cases: one term $\gg$ all others (PS1)
component i dominates when

$$\rho_{\rm tot} \approx \rho_i \gg \rho_{\rm other} \tag{1}$$

- radiation-dominated: $\boxed{a_{\rm rd} \sim t^{1/2}}$

- matter-dominated: $\boxed{a_{\rm md} \sim t^{2/3}}$

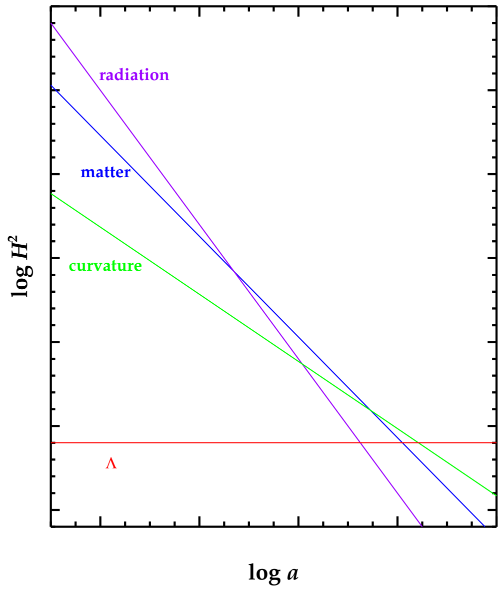
- curvature-dominated $\kappa = -1$; *Q: why?*
  $\boxed{a_{\rm cd} \propto t^{1}}$

- Λ-dominated: $\boxed{a_{\lambda d} \propto e^{+H_\Lambda t}}$

Q: which component most important at early times? late times?

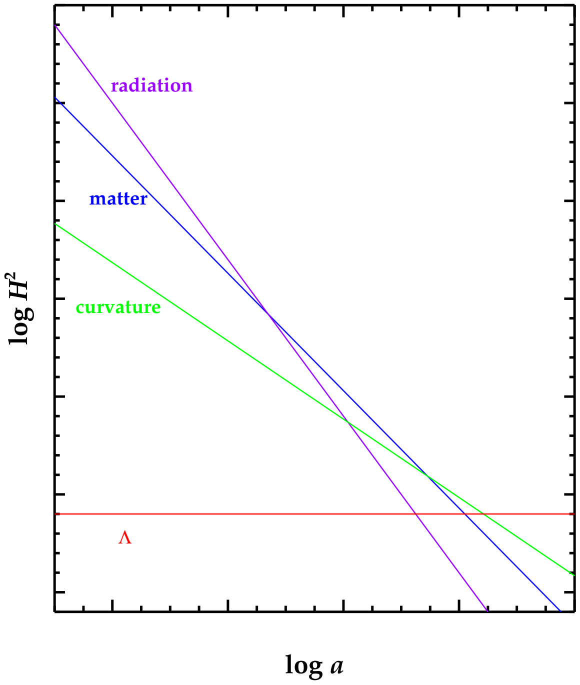
The Cosmic Past

$$
\begin{aligned}
H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\left(\rho_\mathrm{r} + \rho_\mathrm{m} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2} a^{-2} \\
&= \frac{8\pi G}{3}\left(\rho_{\mathrm{r},0} a^{-4} + \rho_{\mathrm{m},0} a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2} a^{-2}
\end{aligned}
$$



Curves for specific choices of parameters $(\rho_{\mathrm{m},0}, \rho_{\mathrm{r},0}, R, \Lambda)$

Q: change if these have different values?



Mix-n-match:

Q: evolution if only matter & rad? Ω?

Q: ... if matter, rad, and curv($\pm$)? Ω?

Q: ... if matter, rad, and Λ? Ω?

Q: ... if matter, rad, curv, and Λ? Ω?

Menu at Al Friedmann's Cosmo Café

Possible Histories of the Universe

Matter + Radiation only: $(\Omega = 1)$

rad-dom → matter-dom; expand forever

Matter + Radiation + Curvature$(-)$: $(\Omega < 1)$

RD → MD → CD; expand forever

Matter + Radiation + Curvature$(+)$: $(\Omega > 1)$

RD → MD → CD → reverse; recollapse

Matter + Radiation + Λ: $(\Omega = 1)$

RD → MD → ΛD: expand forever *exponentially!*

Matter + Radiation + Λ + curv: $(\Omega \neq 1)$

many possibilities! fate depends on detailed composition

Radiation and the Early Universe

note: radiation *always wins out* at early times
$\Rightarrow$ Early U is radiation-dominated
Q: why?

later evolution (which components dominate)
depends on cosmic ingredients
and their relative amounts

Density and Destiny

Enough generalities! What about *our* real Universe?
Fate (and geometry) of U. depend on
current values of $\Omega_{i,0} = \rho_{i,0}/\rho_{\mathrm{crit},0}$
and $\Omega_0 = \sum \Omega_i$ where

$$\begin{aligned}
\rho_{\mathrm{crit},0} &= \frac{3H_0^2}{8\pi G} \\
&= 1.9 \times 10^{-29}\ h^2\ \mathrm{g/cm^{-3}} \approx 10^{-29}\ \mathrm{g/cm^{-3}} \\
&= 2.78 \times 10^{11}\ h^2\ M_\odot\ \mathrm{Mpc^{-3}} \approx 1.4 \times 10^{11}\ M_\odot\ \mathrm{Mpc^{-3}} \\
&\approx 6\ \mathrm{H\,atoms\ m^{-3}}
\end{aligned}$$

Empirical question:

- is $\rho_{\mathrm{tot},0}$ bigger or smaller than this number?
- *density is destiny! weight is fate!*

Cosmic Geometry and Evolution

Consider a universe with $\Omega \neq 1$

Friedmann says

$$\Omega(t) - 1 = \frac{\kappa c^2}{R^2 a^2 H^2} = \frac{\kappa c^2}{R^2 \dot{a}^2} \propto \frac{1}{\dot{a}^2} \tag{2}$$

i.e., Ω *changes* with time

Q: is $|\Omega - 1|$ *increasing or decreasing?*
Q: limiting values of Ω *at large* t*?*
Q: physical interpretation of these limits?
Q: timescale for Ω *to change?*
Q: implications for Ω_0*?*

The Evolution of Ω

Time change of $|\Omega - 1| \propto 1/\dot{a}^2$ is

$$\frac{1}{|\Omega - 1|}\ \frac{d}{dt}|\Omega - 1| = \dot{a}^2 \frac{d}{dt}\frac{1}{\dot{a}^2} \quad (3)$$

$$= -2\ \frac{\ddot{a}/a}{H^2}\ H = 2\ q\ H \quad (4)$$

where *acceleration parameter* $q = -(\ddot{a}/a)/H^2$

Q: why sign choice in q definition?

- generally, $|q| \sim 0.1 - 10$, so
  $|\Omega - 1|$ changes on timescale $1/2|q|H \sim 1/H = t_H \sim t$

- if $\ddot{a} < 0$: ordinary attractive gravity, *decelerating* U
  then $|\Omega - 1|$ *increasing* with time
  $\rightarrow$ Ω driven increasingly away from 1
  Q: unless...?

What is Ω_0?

Procedure 0: Pure Theory

$\Omega = \rho/\rho_{\text{crit}} \sim \rho(t)/H^2(t)$ evolves

- if ever $\Omega = 1$, stays 1 always
- otherwise: $\Omega \rightarrow 0$ or ∞
- physically: expand forever or recollapse
  occurs on cosmic timescale t: current age

$\Omega = 1$ is the only stable value

do the experiment: look around room

$\Omega \neq 0, \infty \rightarrow$ $\Omega = 1$!

else conspiracy: we live just when $\Omega \sim 1$

"Dicke coincidence"

What is Ω_0?

Procedure I: Galaxy Surveys

Goal: measure $\rho_0 \rightarrow$ infer Ω_0

Q: What is $\Omega_{\text{this room}}$*?*
Q: Why can't we use $\rho_{\text{this room}}$*?*
Q: What is needed?
Q: What do galaxy surveys actually measure?
Q: How can we bridge the gap?

Cosmic Density Measurement Procedure I:
Mass-to-Light Ratios

Seems simple...
1. find fair sample of U., with some volume V
2. if measure total mass M, $\to \rho = M/V$

...but telescopes don't measure mass, rather: *luminosity* L
1. find cosmic **luminosity density** $\mathcal{L} = L/V$
2. then find cosmic ratio of mass to luminosity:
   mass-to-light ratio $M/L \equiv \Upsilon$
3. solve for mass density $\rho = \Upsilon\ \mathcal{L}$

Galaxy surveys: $\mathcal{L}_{\rm obs} \sim 2 \times 10^8\ h\ L_\odot\ {\rm Mpc}^{-3}$
...which you will ~verify in PS1!

Need "fair sample" of mass-to-light ratio Υ

Q: how to measure this?

cosmic mass/light sample: galaxies including dark halos

flat rotation curves $v(r) \sim const$

`www: rotation curve`

Newtonian gravity, dynamics apply:
circular motion: $v^2/r \sim g \sim GM_{\rm enclosed}(r)/r^2$
Q: expected behavior for $r >$ visible matter?

Instead: find $v \approx const$ well beyond visible matter
"flat rotation curves"
$\Rightarrow M(r) \sim v^2 r/G \sim r$ for $r \gg r_{\rm vis}$!
dark halo! typically $M_{\rm halo} \sim 5 - 10 M_{\rm vis}$
summing observed light, total dynamical mass:

$$\Upsilon_{\rm halo} \lesssim 25 h M_\odot/L_\odot \to \Omega_{\rm halo} \lesssim 0.02 \ll 1$$

Q: implications? what if this is a fair sample?
Q: why would/wouldn't it be?