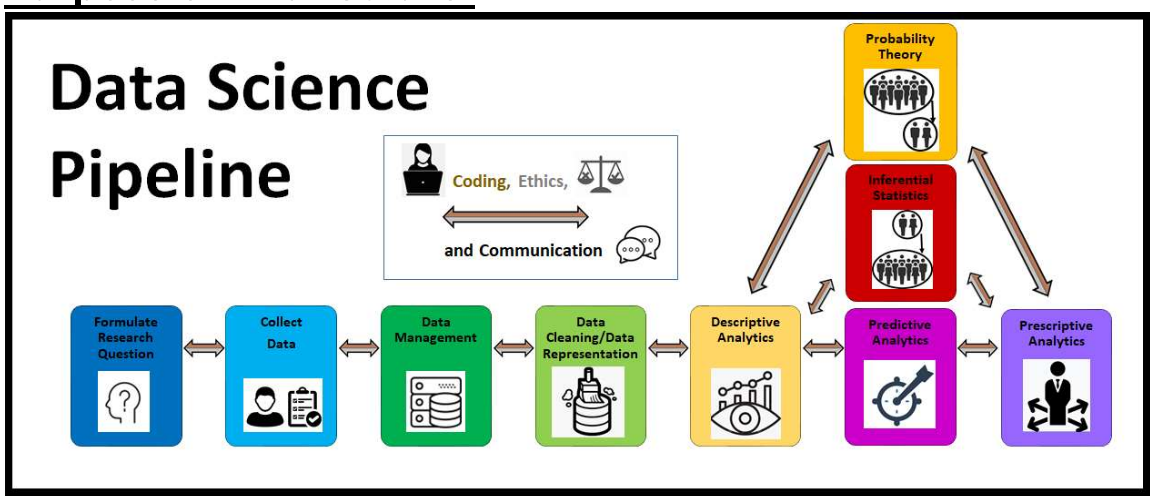
<u>Unit 10 Slides</u>: Introduction to Inference – Hypothesis Testing for Population Means (μ) and Population Proportions (*p)*

<u>Case Studies</u>:

- *<u>COURT CASE</u>: Did the Hamburglar steal cheeseburgers???*
- *Is there sufficient evidence to suggest that the <u>average age of an adult living in the U.S (in 2017)</u> is NOT equal to 50?*
- *Is there sufficient evidence to suggest that the <u>proportion of adults living in the U.S. that approve of the way things are going in the country (in 2017)</u> is NOT equal to 0.75?*

<u>Purpose of this Lecture:</u>



In this lecture we will cover the following **topics in inference.**

1. <u>Two main Types of Inference for Unknown Population Parameters</u>
   1.1. Confidence Intervals
   1.2. Hypothesis Testing
2. <u>Frequentist Hypothesis Testing – Like a Court Case</u>

Additional resources:

Sections 4.3 and 6.1 in Diez, Barr, and Cetinkaya-Rundel, (2015), *OpenIntro Statistics* https://www.openintro.org/download.php?file=os3&redirect=/stat/textbook/os3.php

1. Two Main Types of Inference for Unknown Population Parameters

Suppose we were interested in a population parameter of a large, unknown population. But all we can collect is a random sample from this population.

What we wish we could know: What is the **average age** of ALL adults living in the U.S.?

What we can figure out instead:

1. **Confidence Interval:** What is a plausible range of values for the **average** age of ALL adults living in the U.S.?
2. **Hypothesis Test:** Is there sufficient evidence to suggest some claim about the **average** age of ALL adults living in the U.S.?

What we wish we could know: What is the **proportion** of ALL adults living in the U.S. that are **satisfied** with the way things are going in the country at the time of the survey (2017)?

What we can figure out instead:

1. **Confidence Interval:** What is a plausible range of values for **proportion** of ALL adults living in the U.S. that are **satisfied** with the way things are going in the country at the time of the survey (2017)?
2. **Hypothesis Test:** Is there sufficient evidence to suggest some claim about the **proportion** of ALL adults living in the U.S. that are **satisfied** with the way things are going in the country at the time of the survey (2017)?

2. Frequentist Hypothesis Testing – Like a Court Case

Intuition

In this class, the second type of inference we will make about a large, unknown population is what's known as ___________________ hypothesis testing. While this type of analysis sometimes can seem unintuitive at first, we practice the process of frequentist hypothesis testing in many common decisions that we make, including deciding whether a client is guilty or innocent in a U.S. court case. We will replicate a court case in this section here, introducing the concepts of statistical frequentist inference as we go.

Do you know who this is?

Course Case Preliminary Information

- **Crime:** 10 pounds of cheeseburgers have gone missing from the local McDonald's in the Town of Ronald-McDonald-Land!

- **Defendant:** The Hamburglar has been charged with stealing the burgers.

- **Evidence:** The prosecution presents the following "suspicious" pieces of evidence to the court.
  - Evidence #1: The Hamburglar was found with cheeseburger crumbs on his shirt and mouth.
  - Evidence #2: The Hamburglar's fingerprints were found at the scene of the crime.
  - Evidence #3: Video cameras filmed the Hamburglar walking near the scene of the crime 10 minutes prior to the theft incident.
  - Evidence #4: The Hamburglar has a verifiable record of stealing cheeseburgers from other fastfood restaurants in the past!

- **Trial Proceedings:** The evidence will be assessed by a jury comprised of the ________ STAT207 students in this zoom room today. And they will either:
  - **Decision 1:** Convict the Hamburglar of stealing the burgers (ie. guilty) OR
  - **Decision 2:** Fail to convict the Hamburglar of stealing the burgers (ie. innocent).

- **Sentence:** If found guilty, the defendant will be sentenced to flipping patties for 40 hours.

Initial Assessment from the Jury

Considering just the four suspicious pieces of evidence alone, what is your inclination regarding whether the Hamburglar is guilty or innocent?

a. The Hamburglar is DEFINITELY guilty!

b. The Hamburglar is DEFINITELY innocent!

c. We can never be 100% sure given this evidence alone, but there IS enough suspicious evidence to suggest that he is guilty.

d. We can never be 100% sure given this evidence alone, but there IS NOT enough suspicious evidence to suggest that he is guilty.

e. We can never be 100% sure given this evidence alone. Before making a decision, I'd like to think about all the ways that an INNOCENT defendant could have innocently engaged in these four suspicious pieces of evidence. Then I could use this to get a better assessment of how likely/unlikely it is that an INNOCENT defendant could have had all four of these pieces of evidence against them.

<u>U.S. Law</u>: Two Competing <u>Claims</u> About the Defendant

Claim 1: Client is innocent (not guilty).

Claim 2: Client is guilty.

In frequentist inference terminology, **claim 1** is called the

_______________________, which represents the claim in which

____.

In frequentist inference terminology, **claim 2** is called the

_______________________, which represents the claim in which

____.

<u>U.S. Law</u>: Presumption of Innocence Principle

In U.S. court proceedings we assume that the defendant is __________________ until proven

_____________________.

<u>U.S. Law</u>: Burden of Proof

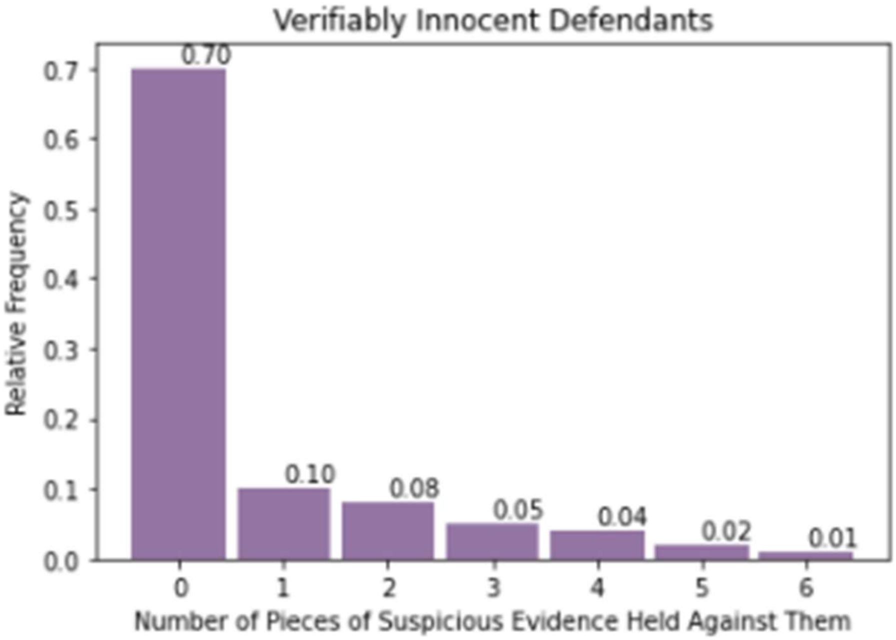
The "burden of proof" rests upon the prosecution. They have the burden of having to

persuade the jury through the evidence that the defendant is ___________________.

If there is not enough evidence, then the defendant is "found" __________________.

Suspicious Evidence Distribution of a Large Population of Innocent Defendants

Suppose the town has a representative, verifiable population of INNOCENT defendants who were accused of the same crime of stealing burgers and has information about how many "suspicious" pieces of evidence that they had against them. The distribution for this data is shown below.



How suspicious is the Hamburglar's evidence?

What is the probability that an innocent defendant had at least 4 suspicious pieces of evidence held against them?

In frequentist inference terminology, the probability that we just calculated is called the ________________.

The smaller the ________________, the more it becomes ______________ that an innocent person (ie. the null hypothesis claim) would have had all this evidence against them. Thus we become ________________ suspicious that the alternative hypothesis is true.

Final Assessment from the Jury

Considering just the four suspicious pieces of evidence alone, what is your decision regarding whether the Hamburglar is guilty or innocent?

a. The Hamburglar is guilty!

b. The Hamburglar is innocent!

c. We can never be 100% sure given this evidence alone, but there IS enough suspicious evidence to suggest that he is guilty.

d. We can never be 100% sure given this evidence alone, but there IS NOT enough suspicious evidence to suggest that he is guilty.

- In frequentist inference terminology, the threshold for this p-value in which we switch our decision (innocent vs. guilty) is called the ___________________________.
- If your p-value is ___________________ the significance level α, you REJECT the baseline claim of innocence, and say there IS enough evidence to suggest the client is guilty.
- If your p-value is ___________________ the significance level α, you FAIL TO REJECT the baseline claim of innocence, and say there IS NOT enough evidence to suggest the client is guilty.

So in the class today:

- The people that said there IS enough evidence to suggest that he is guilty have a significance level that is _______________.

- the people that said there IS NOT enough evidence to suggest that he is guilty have a significance level that is _______________.

What if we set our significance level to be $\alpha = 0.05$ in this case?

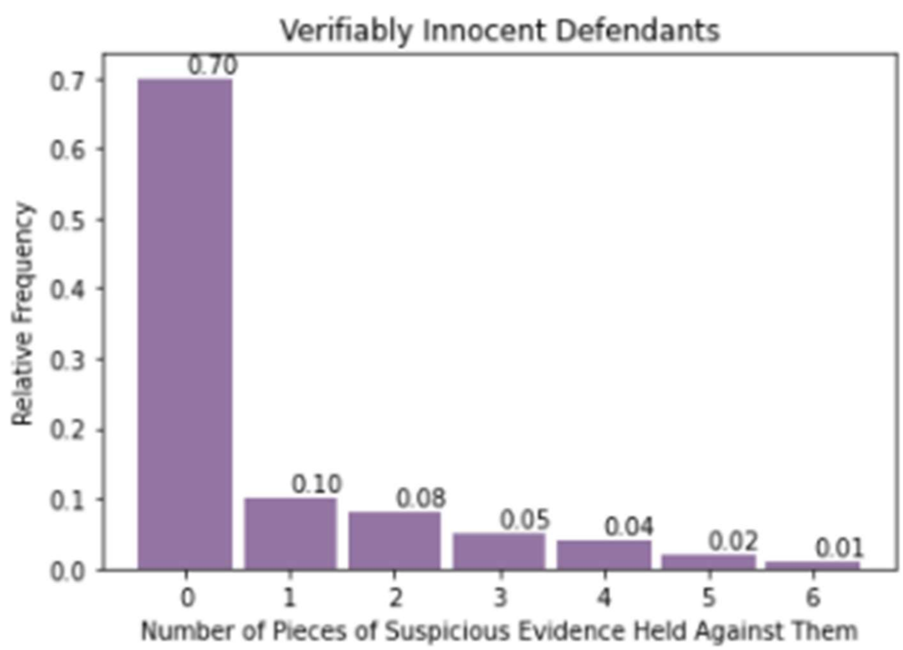
What would everyone's decision need to be?

a. The Hamburglar is guilty!

b. The Hamburglar is innocent!

c. We can never be 100% sure given this evidence alone, but there IS enough suspicious evidence to suggest that he is guilty.

d. We can never be 100% sure given this evidence alone, but there IS NOT enough suspicious evidence to suggest that he is guilty.



If α=0.05 in cases such as this, what percent of INNOCENT defendants will be unfairly sentenced to flipping patties?

- In frequentist inference terminology, the significance level is also called the ______________________, which is the probability of incorrectly rejecting the __________________, given that it was _____________.
- The people that said there IS enough evidence to suggest that he is guilty have a significance level that is _______________. These people are ______________ risk averse to sentencing an innocent defendant to flipping patties for 40 hours.
- the people that said there IS NOT enough evidence to suggest that he is guilty have a significance level that is _______________. These people are ______________ risk averse to sentencing an innocent defendant to flipping patties for 40 hours.

3. Ways to Conduct Frequentist Hypothesis Testing about a Large Unknown Population

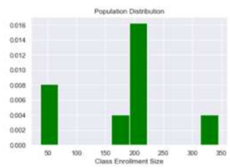
- Way 1: With a p-value

- Way 2: With a test statistic.

- Way 3: With a confidence interval.

4. Properties of Sampling Distributions of Sample Means

Population of Numerical Data



	course	section	enrolled
0	adv307	A	37
1	badm210	A	215
2	badm210	B	178
3	badm210	C	197
4	cs105	A	345
5	cs105	B	201
6	stat107	A	197
7	stat207	A	53

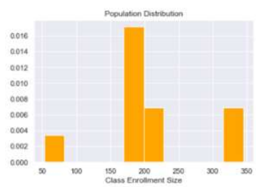
Mean of **Population** Distribution= _________ $= E[X]$

Standard Deviation of **Population** Distribution= _________ $= SD[X]$

The shape of the population distribution and the shape of the sample distribution _____________.

The standard deviation of the population and the standard deviation of the sample are approximately equal when _____________.

Sample of Numerical Data



Random Sample of n=10 Course Enrollments
197
53
345
345
215
178
197
178
215
197

Mean of **Sample** Distribution= _________

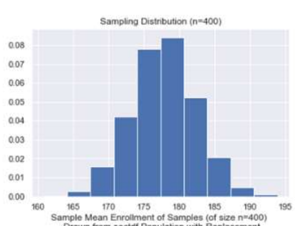
Standard Deviation of **Sample** Distribution= _________

Collect Many Random Samples (all of size n=10) drawn with replacement.

Random Sample of n=10 Course Enrollments *(drawn with replacement from population)*	Random Sample of n=10 Course Enrollments *(drawn with replacement from population)*	Random Sample of n=10 Course Enrollments *(drawn with replacement from population)*	...	Random Sample of n=10 Course Enrollments *(drawn with replacement from population)*
197	215	53	...	215
37	215	53	...	197
345	53	197	...	37
201	201	53	...	215
178	53	197	...	197
37	345	197	...	197
53	197	178	...	345
201	37	215	...	345
201	201	197	...	197
197	37	197	...	201

$\bar{x}$ $\bar{x}$ $\bar{x}$ $\bar{x}$

Sampling Distribution of Sample Means



Sample Means
164.7
155.4
153.7
...
214.6

Need to know the following to make an inference about *Unknown* Population Mean μ:

Mean of **Sampling** Distribution $\approx E[\bar{X}] =$ _______

Standard deviation of **Sampling** Distribution $\approx SD[\bar{X}] =$ _______

Shape of **Sampling** Distribution is ________________ when either:

1. ______________________ OR
2. ______________________________________.

5. General Frequentist Hypothesis Testing – Using p-values

1. **Set up two hypotheses.**
   a. **Null hypothesis:** represents the claim:
      i. "nothing is happening"
      ii. "status quo"
   b. **Alternative hypothesis:** represents the claim:
      i. "something is happening"
      ii. "claim that you are trying to test"

2. **Generate a Sampling Distribution (of Sample Statistics) from a Population that ASSUMES the Null Hypothesis is True**

   This can be done:
   i. Theoretically
   ii. Or Via Simulation

3. **Collect an Observed Sample Statistic:**

   Collect an ACTUAL random sample from the population you'd like to conduct inference on and calculate a sample statistic.

4. **Calculate the p-value**

$$p-value = P\left(\begin{matrix} sample\ statistic\ that\ is\ at\ least \\ as\ suspicious\ (in\ favor\ of\ the\ alternative \\ hypotheis)\ as\ the\ observed\ sample\ statistic \end{matrix} \mid Null\ hypothesis\ is\ True\right)$$

5. **Make a Decision**
   a. If $\mathbf{p-value} < \boldsymbol{\alpha}$**,** then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."
   b. If $\mathbf{p-value} \geq \boldsymbol{\alpha}$, then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

6. Frequentist Hypothesis Testing: for a Population Mean μ - Using p-values

Preliminary Example - Intuition

Ex: Suppose that in 2007 the average age of all adults living in the U.S. was thought to have been 50. In 2017, researchers are skeptical that that is still the case. We would like to conduct hypothesis testing to test these claims.

So in 2017, we collect a random sample of size 1489 with an average of 50.49 years and a standard deviation of 17.84 years. **Suppose we also know that the standard deviation of ALL adults living in the U.S. in 2017 is $\boldsymbol{\sigma = 18}$.**

See Unit 10 notebook (section 6) for code involved in this example.

1. Set up your hypotheses for this test.

2. Are the Central Limit Theorem Conditions (for Sample Means) met by this sample?

3. If so, describe what we know about the sampling distribution of sample means (of random samples of size n=1489) so far. Describe what we know about the distribution of z-scores of the sample means in our sampling distribution so far.

4. If we assume the null hypothesis is true, what other information can we add/assume about our sampling distribution of sample means (of random samples of size n=1489) so far?

5. What *types* of sample means would make us "suspicious" that our alternative hypothesis is true?

6. Calculate the p-value for this hypothesis test.

$$p - value = P\left(\begin{array}{c} sample\ statistic\ that\ is\ at\ least \\ as\ suspicious\ (in\ favor\ of\ the\ alternative \\ hypotheis)\ as\ the\ observed\ sample\ statistic \end{array} \middle|\ Null\ hypothesis\ is\ True\right)$$

$$= P\left(\begin{array}{c} \_\_\_\_\_\_\_\_\_\_\_\_\ that\ is\ at\ least \\ as\ suspicious\ (in\ favor\ of\ the\ alternative \\ hypotheis)\ as\ \_\_\_\_\_\_\_\_\_\_\_\_ \end{array} \middle|\ Null\ hypothesis\ is\ True\right)$$

$$= P(\_\_\_\_ \geq \_\_\_\_\ OR\ \_\_\_\_ \leq \_\_\_\_)$$

$$= P(\_\_\_\_ \geq \_\_\_\_) + P(\_\_\_\_ \leq \_\_\_\_)$$

$$= \_\_\_\_ * P(\_\_\_\_ \geq \_\_\_\_)$$

= ________________________

Assuming that **Ho**: μ = _________

7. Using a significance level of $\alpha = 0.05$, make a conclusion about your hypotheses.

7. Frequentist Hypothesis Testing: for a Population Mean μ - Using a p-value

Testing Claim: $\boldsymbol{\mu \neq \mu_0}$

1. **Set up two hypotheses.**
   a. **Null hypothesis:**
      $H_0: \mu = \mu_0$
   b. **Alternative hypothesis:**
      $H_A: \mu \neq \mu_0$
2. **Check the CLT Conditions (for Sample Means)**

   If they hold, then you can proceed with the hypothesis testing. If not, you may be making an invalid inference and a conclusion that is based on flawed reasoning.

3. **Collect an Observed Sample Statistic:**
   Collect an ACTUAL random sample from the population you'd like to conduct inference on and calculate the sample mean $\bar{x}_0$.

4. **Calculate the p-value**

   One way: $p-value = \begin{Bmatrix} 2P(\bar{X} \geq \bar{x}_0), & if\ \bar{x}_0 \geq \mu_0 \\ 2P(\bar{X} \leq \bar{x}_0), & if\ \bar{x}_0 \leq \mu_0 \end{Bmatrix}$,
   assuming $\bar{X} \sim N(mean = \_\_\_\_\_\_\_, std\ deviation = \_\_\_\_\_\_\_\_\_)$

   We call $\frac{\bar{x}_0 - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ the **test statistic** for this hypothesis test.

   Another way: $p-value = 2P(Z \geq |\frac{\bar{x}_0 - \mu_0}{\frac{\sigma}{\sqrt{n}}}|)$,
   assuming $Z \sim N(mean = \_\_\_\_\_\_\_, std\ deviation = \_\_\_\_\_\_\_\_\_)$

5. **Make a Decision**

a. If $\mathbf{p-value}<\boldsymbol{\alpha},$ then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."

b. If $\mathbf{p-value}\geq\boldsymbol{\alpha}$, then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

8. What do you do if you don't know σ?

If we don't know μ, it is quite often the case that we don't know σ either.

Useful property:

As n increases, $s=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})}{n-1}}$ gets closer to σ.

"Rule of Thumb": How large does n have to be to approximate $s\approx\sigma$?

What to do: Plug in s for σ when __________________.

Ex: Suppose that in 2007 the average age of all adults living in the U.S. was thought to have been 50. In 2017, researchers are skeptical that that is still the case. We would like to conduct hypothesis testing to test these claims.

So in 2017, we collect a random sample of size 1489 with an average of 50.49 years and a standard deviation of 17.84 years. **Suppose we do not know the standard deviation of ALL adults living in the U.S.** Conduct another hypothesis test.

9. General Frequentist Hypothesis Testing – Using Test Statistics

1. **Set up two hypotheses.**
   a. **Null hypothesis:** represents the claim:
      i. “nothing is happening”
      ii. “status quo”
   b. **Alternative hypothesis:** represents the claim:
      i. “something is happening”
      ii. “claim that you are trying to test”

2. **Generate a Sampling Distribution (of Sample Statistics) from a Population that ASSUMES the Null Hypothesis is True**

   This can be done:
   i. Theoretically
   ii. Or Via Simulation

3. **Collect an Observed Sample Statistic:**

   Collect an ACTUAL random sample from the population you’d like to conduct inference on and calculate a sample statistic.

4. **Calculate the Test Statistic for *this* Hypothesis Test**

   How to calculate this differs based on the test.

5. **Make a Decision**

   Changes based on the test.

10. Frequentist Hypothesis Testing: for a Population Mean μ - Using a Test Statistic

Testing Claim: $\boldsymbol{\mu \neq \mu_0}$

1. **Set up two hypotheses.**
   a. **Null hypothesis:**
      $H_0: \mu = \mu_0$
   b. **Alternative hypothesis:**
      $H_A: \mu \neq \mu_0$
2. **Check the CLT Conditions (for Sample Means)**

   If they hold, then you can proceed with the hypothesis testing. If not, you may be making an invalid inference and a conclusion that is based on flawed reasoning.

3. **Collect an Observed Sample Statistic:**
   Collect an ACTUAL random sample from the population you'd like to conduct inference on and calculate the sample mean $\bar{x}_0$.

4. **Calculate the Test Statistic for *this* Hypothesis Test**
   We call $Z_0 = \frac{\bar{x}_0 - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ the **test statistic** for this hypothesis test.

5. **Make a Decision**
   a. If $|\textbf{test statistic}| > \mathbf{Z}_{\mathbf{1-\frac{\alpha}{2}}}$**,** then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."
   b. If $|\textbf{test statistic}| \leq \mathbf{Z}_{\mathbf{1-\frac{\alpha}{2}}}$, then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

$\mathbf{Z}_{\mathbf{1-\frac{\alpha}{2}}}$ **represents ________________________________**

Ex: Suppose that in 2007 the average age of all adults living in the U.S. was thought to have been 50. In 2017, researchers are skeptical that that is still the case. We would like to conduct hypothesis testing to test these claims.

So in 2017, we collect a random sample of size 1489 with an average of 50.49 years and a standard deviation of 17.84 years. **Suppose we do not know the standard deviation of ALL adults living in the U.S.** Conduct another hypothesis test using the test statistic to make a decision.

11. General Frequentist Hypothesis Testing – Using Confidence Intervals

Null value θ_0

1. **Set up two hypotheses.**
   a. $\boldsymbol{H_0}: population\ parameter = \theta_0$
   b. $\boldsymbol{H_A}: population\ parameter \neq \theta_0$

2. **Generate a Confidence Interval for the Population Parameter**
   a. Make sure the conditions for making this confidence interval are met first!

3. **Make a Decision**
   a. If $\boldsymbol{\theta_0}$ is __________________ the confidence interval, then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."

   b. If $\boldsymbol{\theta_0}$ is __________________ the confidence interval, then then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

Ex: Suppose that in 2007 the average age of all adults living in the U.S. was thought to have been 50. In 2017, researchers are skeptical that that is still the case. We would like to conduct hypothesis testing to test these claims.

So in 2017, we collect a random sample of size 1489 with an average of 50.49 years and a standard deviation of 17.84 years. Suppose we don't know the standard deviation of ALL adults living in the U.S.

Conduct your hypothesis test using a 95% confidence interval.

12. Properties of Sampling Distributions of Sample Proportions

Population of Categorical Data

	toss	value
0	heads	1
1	tails	0

Population Proportion: **p**

Sample of Categorical Data

	Random Sample
0	1
1	0
1	0
0	1
0	1
1	0
1	0
1	0
0	1
0	1

Sample Proportion: $\hat{p}$

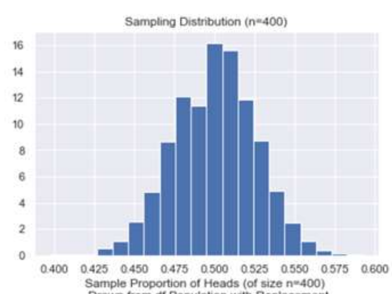
Collect Many Random Samples (all of size n=10) drawn with replacement.

Random Sample of n=10 Tosses *(drawn with replacement from population)*	Random Sample of n=10 Tosses *(drawn with replacement from population)*	Random Sample of n=10 Tosses *(drawn with replacement from population)*	...	Random Sample of n=10 Tosses *(drawn with replacement from population)*
1	1	1	...	0
0	1	0	...	0
1	1	1	...	0
0	1	0	...	0
1	1	0	...	1
0	1	0	...	0
0	1	1	...	0
1	1	1	...	0
0	1	0	...	1
1	0	0	...	1

$\hat{p}$ $\hat{p}$ $\hat{p}$ $\hat{p}$

Sampling Distribution of Sample Proportions

Sample Proportions
0.5
0.9
0.4
...
0.3



Need to know the following to make an inference about *Unknown* Population Proportion *p*:

Mean of **Sampling** Distribution $\approx E[\hat{p}] =$ ________

Standard deviation of **Sampling** Distribution $\approx SD[\hat{p}] =$ ________

Shape of **Sampling** Distribution is ____________________ when:

____.

13. Frequentist Hypothesis Testing: for a Population Proportion p - Using p-values

Preliminary Example - Intuition

Ex: Suppose that a politician claims that 75% of all adults living in the U.S. (in 2017) are satisfied with the way things are going in the country. A team of researchers is skeptical of this claim and would like to test it. They collect a random sample of size n=1435 that has a sample proportion of 0.301 that approve.

1. Set up your hypotheses for this test.

2. Are the Central Limit Theorem Conditions (for Sample Proportions) met by this sample?

3. If so, describe what we know about the sampling distribution of sample proportions (of random samples of size n=1435) so far. Describe what we know about the distribution of z-scores of the sample proportions in our sampling distribution so far.

4. If we assume the null hypothesis is true, what other information can we add/assume about our sampling distribution of sample proportions (of random samples of size n=1435) so far?

5. What *types* of sample proportions would make us "suspicious" that our alternative hypothesis is true?

6. Calculate the p-value for this hypothesis test.

$$p - value \quad = P\begin{pmatrix} \text{sample statistic that is at least} \\ \text{as suspicious (in favor of the alternative} \\ \text{hypotheis) as the observed sample statistic} \end{pmatrix} \Big| \text{Null hypothesis is True}$$

$$= P\begin{pmatrix} \text{\_\_\_\_\_\_\_\_\_\_\_\_ that is at least} \\ \text{as suspicious (in favor of the alternative} \\ \text{hypotheis) as \_\_\_\_\_\_\_\_\_\_\_\_} \end{pmatrix} \Big| \text{Null hypothesis is True}$$

$$= P(\_\_\_\_ \geq \_\_\_\_ \; OR \; \_\_\_\_ \leq \_\_\_\_)$$

$$= P(\_\_\_\_ \geq \_\_\_\_) + P(\_\_\_\_ \leq \_\_\_\_)$$

$$= \_\_\_\_ * P(\_\_\_\_ \leq \_\_\_\_)$$

= ______________________

7. Use a significance level of $\alpha = 0.10$, make a conclusion about your hypotheses.

14. Frequentist Hypothesis Testing: for a Population Proportion p - Using a p-value

Testing Claim: $p \neq p_0$

1. **Set up two hypotheses.**
   a. **Null hypothesis:**
      $H_0: p = p_0$
   b. **Alternative hypothesis:**
      $H_A: p \neq p_0$

2. **Check the CLT Conditions (for Sample Proportions)**

   If they hold, then you can proceed with the hypothesis testing. If not, you may be making an invalid inference and a conclusion that is based on flawed reasoning. **(Substitute p_0 for p when checking these conditions).**

3. **Collect an Observed Sample Statistic:**
   Collect an ACTUAL random sample from the population you'd like to conduct inference on and calculate the sample proportion $\hat{p}_0$.

4. **Calculate the p-value**
   One way: $p - value = \begin{cases} 2P(\hat{p} \geq \hat{p}_0), & if\ \hat{p}_0 \geq p_0 \\ 2P(\hat{p} \leq \hat{p}_0), & if\ \hat{p}_0 \leq p_0 \end{cases}$,
   assuming $\hat{p} \sim N(mean =$ _______, $std\ deviation =$ _________)

   We call $\frac{\hat{p}_0 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ the **test statistic** for this hypothesis test.

   Another way: $p - value = 2P(Z \geq |\frac{\hat{p}_0 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}|)$,
   assuming $Z \sim N(mean =$ _______, $std\ deviation =$ _________)

5. Make a Decision

a. If $\mathbf{p-value} < \boldsymbol{\alpha}$**,** then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."

b. If $\mathbf{p-value} \geq \boldsymbol{\alpha}$, then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

15. Frequentist Hypothesis Testing: for a Population Mean p - Using a Test Statistic

Testing Claim: $p \neq p_0$

1. **Set up two hypotheses.**
   a. **Null hypothesis:**
      $H_0: p = p_0$
   b. **Alternative hypothesis:**
      $H_A: p \neq p_0$
2. **Check the CLT Conditions (for Sample Proportions)**

   If they hold, then you can proceed with the hypothesis testing. If not, you may be making an invalid inference and a conclusion that is based on flawed reasoning. **(Substitute p_0 for p when checking these conditions).**

3. **Collect an Observed Sample Statistic:**
   Collect an ACTUAL random sample from the population you'd like to conduct inference on and calculate the sample proportion $\hat{p}_0$.

4. **Calculate the Test Statistic for *this* Hypothesis Test**
   We call $Z_0 = \frac{\hat{p}_0 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ the **test statistic** for this hypothesis test.

5. **Make a Decision**
   a. If $|\textbf{test statistic}| < \mathbf{Z_{1-\frac{\alpha}{2}}}$**,** then we "reject the null hypothesis." And we say that "there IS sufficient evidence to suggest the alternative hypothesis."
   b. If $|\textbf{test statistic}| \geq \mathbf{Z_{1-\frac{\alpha}{2}}}$, then we "fail to reject the null hypothesis." And we say that "there IS NOT sufficient evidence to suggest the alternative hypothesis."

$\mathbf{Z_{1-\frac{\alpha}{2}}}$ **represents** ________________________________

Ex: Suppose that a politician claims that 75% of all adults living in the U.S. (in 2017) are satisfied with the way things are going in the country. A team of researchers is skeptical of this claim and would like to test it. They collect a random sample of size n=1435 that has a sample proportion of 0.301 that approve.

a. Set up your hypotheses.

b. Check your conditions for conducting inference.

c. Make a conclusion using a p-value and a significance level of 0.10.

d. Make a conclusion using a test statistic and a significance level of 0.01.

16. Frequentist Hypothesis Testing: for a Population Mean p - Using a Confidence Interval

Testing Claim: $p \neq p_0$

(Substitute p_0 for p when checking these conditions and calculating your standard error).

e. Make a conclusion using a 90% confidence interval.