Unit 17: Classification Accuracy of Classifier Models

Case Study: Modelling Approval for the President's Foreign Policy with Age and Sex

Model 1 In the beginning of this analysis we will model the following response variable with the following explanatory variable.

- response: approval of the president's foreign policy (approve vs. disapprove)
    - age

In attempt to improve our classification accuracy of the previous model, we will introduce a new model with more explanatory **Model 2**

- response: approval of the president's foreign policy (approve vs. disapprove)
- explanatory:
    - sex
    - age
    - party

Will model 2 outperform model 1 in terms of classification accuracy? How do we measure how well each of these models classified the observations in the sample?

New Package Installation

New package: scikit-learn - machine learning package

To install this on your computer enter the following command from a terminal or anaconda window:

```
conda install scikit-learn
```

In [1]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt

import statsmodels.api as sm
import statsmodels.formula.api as smf
```

1. How to predict the response variable of a given observation using a logistic regression model.

See Unit 17 slides (Section 1)

2. How can we use a predictive probability to classify a given observation using a logistic regression model.

See Unit 17 slides (section 2)

Let's again examine our random sample of adults living in the U.S. (from 2017) from Pew Research. We will just use three variables for this analysis, so we will just create a dataframe using these three variables.

In [2]:
```
missing_values = ["NaN", "nan", "Don't know/Refused (VOL.)"]
#
df = pd.read_csv('Feb17public.csv', na_values=missing_values)[['age','sex','q5cf1']]
df.head()
```

Out[2]:

	age	sex	q5cf1
0	80.0	Female	NaN
1	70.0	Female	Disapprove
2	69.0	Female	Disapprove
3	50.0	Male	NaN
4	70.0	Female	Disapprove

In [3]:
```
df.shape
```

Out[3]: (1503, 3)

In [4]:
```
df=df.dropna()
df.head()
```

Out[4]:

	age	sex	q5cf1
1	70.0	Female	Disapprove
2	69.0	Female	Disapprove
4	70.0	Female	Disapprove
6	89.0	Female	Disapprove
7	92.0	Female	Approve

In [5]:
```
df.shape
```

Out[5]: (691, 3)

In [6]:
```
df['y']=df['q5cf1'].map({'Disapprove':0,'Approve':1})
df.head()
```

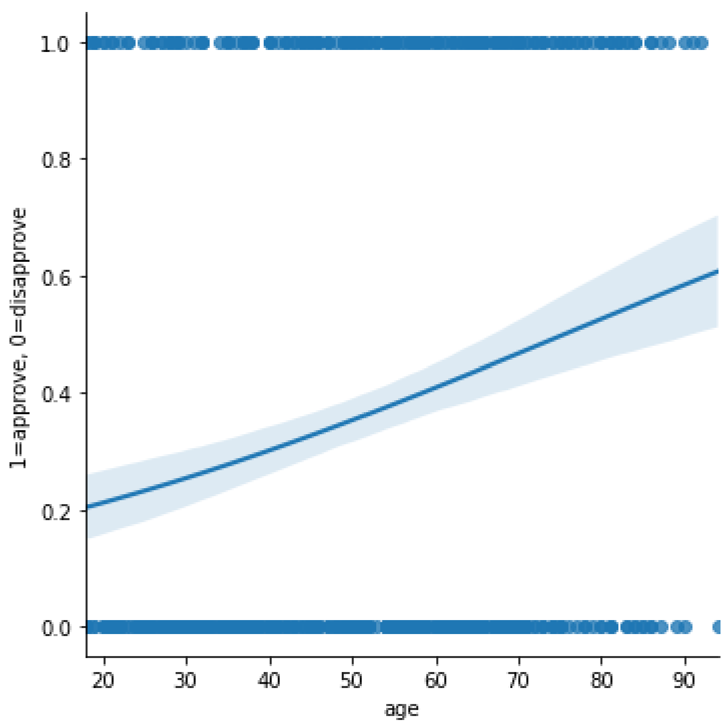
Out[6]:

	age	sex	q5cf1	y
1	70.0	Female	Disapprove	0
2	69.0	Female	Disapprove	0
4	70.0	Female	Disapprove	0
6	89.0	Female	Disapprove	0
7	92.0	Female	Approve	1

2.1. Let's first fit a logistic regression model to predict the probability of a person in the sample approving of the president's foreign policy, given age.

Because we only have an explanatory variable (age) and a response variable (y), we can plot the logistic regression curve over the scatterplot of these observations in the sample.

In [7]:
```
sns.lmplot(x="age", y='y',data=df, logistic=True)
plt.ylabel('1=approve, 0=disapprove')
plt.show()
```



The equation for this logistic regression model is the following:

$$log(\frac{\hat{p}}{1-\hat{p}}) = -1.7872 + 0.0236(age).$$

And we can rewrite this equation in another way as well to directly show us the predictive probability for a given age:

$$\hat{p} = \frac{e^{-1.7872+0.0236(age)}}{e^{-1.7872+0.0236(age)}+1}$$

In [8]:
```
mod1 = smf.logit(formula='y ~ age', data=df).fit()
mod1.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.633703
         Iterations 5
```

Out[8]:

Logit Regression Results

Dep. Variable:	y	**No. Observations:**	691
Model:	Logit	**Df Residuals:**	689
Method:	MLE	**Df Model:**	1
Date:	Thu, 08 Apr 2021	**Pseudo R-squ.:**	0.03047
Time:	13:28:06	**Log-Likelihood:**	-437.89
converged:	True	**LL-Null:**	-451.65
Covariance Type:	nonrobust	**LLR p-value:**	1.553e-07

	coef	std err	z	P>|z|	[0.025	0.975]
Intercept	-1.7872	0.254	-7.023	0.000	-2.286	-1.288
age	0.0236	0.005	5.128	0.000	0.015	0.033

2.2. What is the predictive probability of a 70-year-old approving of the president's foreign policy (using this fitted logistic regression model)?

In [9]:
```
mod1.predict(exog=dict(age=70))
```

Out[9]:
```
0    0.466859
dtype: float64
```

The probability that a 70 year old approves of the president's foreign policy (in this random sample of adults living in the U.S. in 2017) is 0.466859.

2.3. If we use a predictive probability threshold of $p_0 = 0.5$ for this given logistic regression model (ie. classifier model), what opinion will we classifiy this 70-year-old as having?

Because $0.466859 \leq p_0 = 0.5$, we classify this person with y=0 (or that they disapprove of the president's foreign policy).

2.4 Using this predictive probability threshold of p_0=0.5, what is the minimum age that a person needs to be in order to be classified as approving the president's foreign policy in this sample?

What value of age satisfies this inequality? $\hat{p} = \frac{e^{-1.7872+0.0236(age)}}{e^{-1.7872+0.0236(age)}+1} > 0.5.$

$log(\frac{0.5}{1-0.5}) < -1.7872 + 0.0236(age).$

$age > 75.7288.$

In [10]:
```
#Plug in 0.5 into your logistic regression equation and solve for age.
threshold_age=(np.log(.5/(1-.5))+1.7872)/.0236
threshold_age
```

Out[10]:
```
75.72881355932203
```

3. Types of Misclassifications and Correct Classifications

3.1. Let's classify ALL of the observations in the sample using this predictive probability threshold of p_0 and the logistic regression model.

First we need to get the predictive probabilities for all of the observations in our df dataframe.

We can use the **.predict()** function and simply just input the whole dataframe into the **dict()** function for the **exog** parameter. While the df dataframe has more columns than just **age** in it (ie. our only explanatory variable at the moment in this logistic regression model) the predict function is intelligent enough to know that we only want to extract just the columns values that correspond to the given logistic regression model (mod1).

We add these predictive probabilities as a column in our dataframe.

In [11]:
```
pred_probabilities=mod1.predict(exog=dict(df))
df['predictive_prob']=pred_probabilities
df
```

Out[11]:

	age	sex	q5cf1	y	predictive_prob
1	70.0	Female	Disapprove	0	0.466859
2	69.0	Female	Disapprove	0	0.460982
4	70.0	Female	Disapprove	0	0.466859
6	89.0	Female	Disapprove	0	0.578423
7	92.0	Female	Approve	1	0.595610
...	...	...	...	...	...
1494	23.0	Female	Approve	1	0.223812
1498	37.0	Male	Approve	1	0.286448
1499	30.0	Female	Approve	1	0.253858
1501	67.0	Male	Disapprove	0	0.449260
1502	35.0	Female	Approve	1	0.276884

691 rows × 5 columns

Then we can create another column in our dataframe that is our classification (or predicted value for y) for each of the observations. To do this we can set up a row condition on the inside of some parantheses.

This (row condition for a dataframe) by itself will produce a series of True or False values depending on whether the condition for that particular row is true or not.

Then by using 1*(row condition for a dataframe), we translate each True to a 1 and each False to a 0.

In [12]:
```
df['yhat']=1*(df['predictive_prob']>0.5)
df
```

Out[12]:

	age	sex	q5cf1	y	predictive_prob	yhat
1	70.0	Female	Disapprove	0	0.466859	0
2	69.0	Female	Disapprove	0	0.460982	0
4	70.0	Female	Disapprove	0	0.466859	0
6	89.0	Female	Disapprove	0	0.578423	1
7	92.0	Female	Approve	1	0.595610	1
...	...	...	...	...	...	...
1494	23.0	Female	Approve	1	0.223812	0
1498	37.0	Male	Approve	1	0.286448	0
1499	30.0	Female	Approve	1	0.253858	0
1501	67.0	Male	Disapprove	0	0.449260	0
1502	35.0	Female	Approve	1	0.276884	0

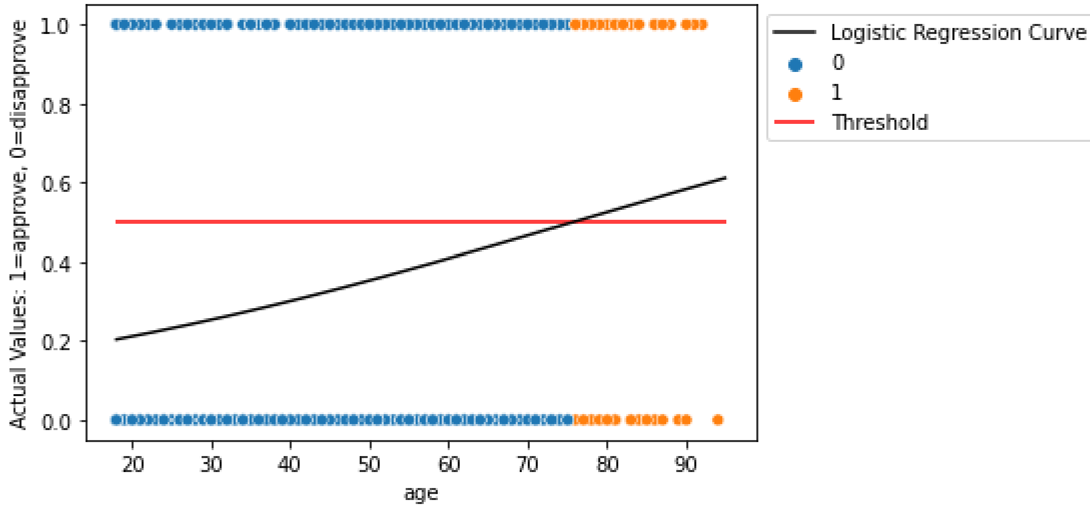
691 rows × 6 columns

3. Types of Misclassifications and Correct Classifications

See Unit 17 slides (section 3).

3.1 Let's plot our predicted (classified) y-values for the observations (shown in blue and orange) as well as our actual values for y (shown on the y-axis.)

In [13]:
```python
x=np.arange(18,95,.1)
p=np.exp(-1.7872+0.0236*x)/(1+np.exp(-1.7872+0.0236*x))
sns.scatterplot(x="age", y='y',data=df, hue='yhat')
plt.plot(x,p, color='black', label='Logistic Regression Curve')
plt.hlines(y=0.5, xmin=18, xmax=95, color='red', label='Threshold')
plt.ylabel('Actual Values: 1=approve, 0=disapprove')
plt.legend(bbox_to_anchor=(1,1))
plt.show()
```



3.2. Next, let's create a confusion matrix for this classifier model and the given threshold.

In [14]:
```python
# This import requires that you already
# installed the scikit-learn library
# as described in the introduction to this chapter.
#
from sklearn.metrics import confusion_matrix, roc_curve, roc_auc_score
```

In [15]:
```python
confusion_matrix(y_true=df['y'], y_pred=df['yhat'])
```

Out[15]:
```
array([[420,  22],
       [221,  28]], dtype=int64)
```

In [16]:
```python
tn, fp, fn, tp = confusion_matrix(y_true=df['y'],
                                  y_pred=df['yhat']).ravel()
(tn, fp, fn, tp)
```

Out[16]:
```
(420, 22, 221, 28)
```

3.3. What is the number of true positive, false positive, true negative, and false negative observations in the sample using the logistic regression model and the predictive threshold of $p_0 = 0.5$?

3.4 What is the sensitivity of the logistic regression model with the predictive threshold of $p_0 = 0.5$ using the sample data?

In [17]:
```
sensitivity=tp/(tp+fn)
print('sensitivity:', sensitivity)
```

```
sensitivity: 0.11244979919678715
```

This is quite low, which indicates that the logistic regression model with $p_0 = 0.05$ did not do well at correctly classifying people that actually approved of the president's foreign policy.

3.5 What is the specificity of the logistic regression model with the predictive threshold of $p_0 = 0.5$ using the sample data?

In [18]:
```
specificity=tn/(tn+fp)
print('specificity:', specificity)
```

```
specificity: 0.9502262443438914
```

This is quite high, which indicates that the logistic regression model with $p_0 = 0.05$ did a pretty good job at correctly classifying people that actually disapproved of the president's foreign policy.

3.6 What is the false positive rate of the logistic regression model with the predictive threshold of $p_0 = 0.5$ using the sample data?

In [19]:
```
false_positive_rate=fp/(tn+fp)
print('false positive rate:', false_positive_rate)
```

```
false positive rate: 0.049773755656108594
```

This is quite low, which indicates that the logistic regression model with $p_0 = 0.05$ did a pretty good job at correctly classifying people that actually disapproved of the president's foreign policy.

4. Relationship between Sensitivity and Specificity

See Unit 16 slides (Section 4)

4.1 Using the same logistic regression model as before, and now a predictive probability threshold of $p_0 = 0.3$ to classify all of the observations in the sample.

Let's first re-classify our observations in our dataframe with our new threshold of $p_0 = 0.3$.

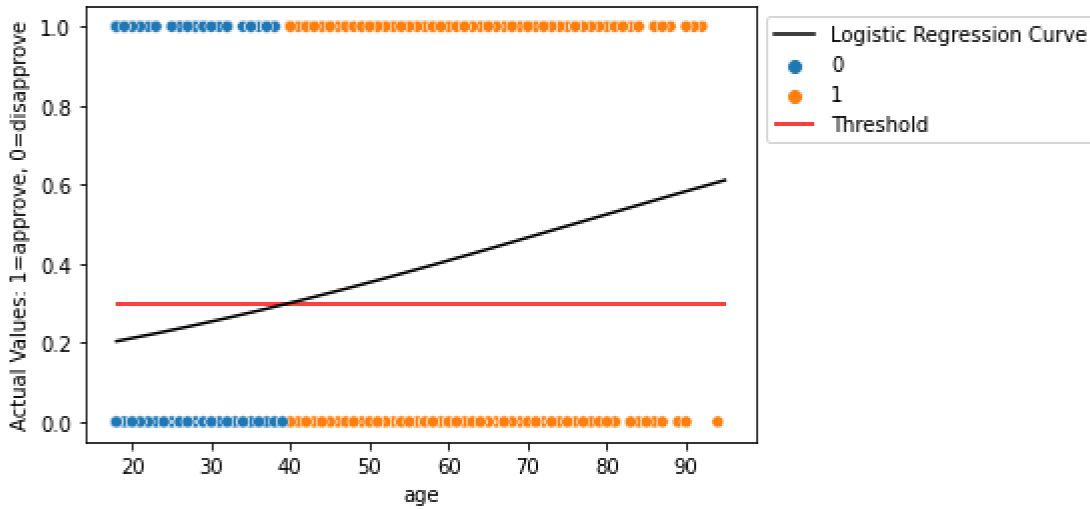
In [20]:
```
df['yhat']=1*(df['predictive_prob']>0.3)
df
```

Out[20]:

	age	sex	q5cf1	y	predictive_prob	yhat
1	70.0	Female	Disapprove	0	0.466859	1
2	69.0	Female	Disapprove	0	0.460982	1
4	70.0	Female	Disapprove	0	0.466859	1
6	89.0	Female	Disapprove	0	0.578423	1
7	92.0	Female	Approve	1	0.595610	1
...	...	...	...	...	...	...
1494	23.0	Female	Approve	1	0.223812	0
1498	37.0	Male	Approve	1	0.286448	0
1499	30.0	Female	Approve	1	0.253858	0
1501	67.0	Male	Disapprove	0	0.449260	1
1502	35.0	Female	Approve	1	0.276884	0

691 rows × 6 columns

In [21]:
```
x=np.arange(18,95,.1)
p=np.exp(-1.7872+0.0236*x)/(1+np.exp(-1.7872+0.0236*x))
sns.scatterplot(x="age", y='y',data=df, hue='yhat')
plt.plot(x,p, color='black', label='Logistic Regression Curve')
plt.hlines(y=0.3, xmin=18, xmax=95, color='red', label='Threshold')
plt.ylabel('Actual Values: 1=approve, 0=disapprove')
plt.legend(bbox_to_anchor=(1,1))
plt.show()
```



4.2. Find the sensitivity and specificity for this threshold and classifier.

In [22]:
```
tn, fp, fn, tp = confusion_matrix(y_true=df['y'],
                                  y_pred=df['yhat']).ravel()
(tn, fp, fn, tp)
```

Out[22]: (172, 270, 51, 198)

In [23]:
```
sensitivity=tp/(tp+fn)
print('sensitivity:', sensitivity)
```

sensitivity: 0.7951807228915663

In [24]:
```
specificity=tn/(tn+fp)
print('specificity:', specificity)
```

specificity: 0.3891402714932127

4.3 Which predictive probability threshold yielded better results? $p_0 = .5$ or $p_0 = 0.3$?

Ideally, we would like both our sensitivty and specificity to be high. However, we know that generally when one of these increases, the other will decrease (based on us changing our predictive probability threshold).

Which of these results is best is dependent on how much we care accurately classifying people that actually approve vs. accurately classifying people that disapprove.

- If we care more about accurately classifying people that actually approve (ie. actually have y=1), we would want to pick the threshold of $p_0 = 0.3$ that yields a higher sensitivity of 0.795.
- If we care more about accurately classifying people that actually disapprove (ie. actually have y=0), we would want to pick the threshold of $p_0 = 0.5$ that yields a higher specificity of 0.950.

5. ROC and AUC Which *classifier* will give us the best combinations of (false positive rate, true positive rate) for all sets of thresholds?

See unit 17 slides (section 5)

5.1Model 1: Logistic regression model that predicts approval of presidential foreign policy given age.

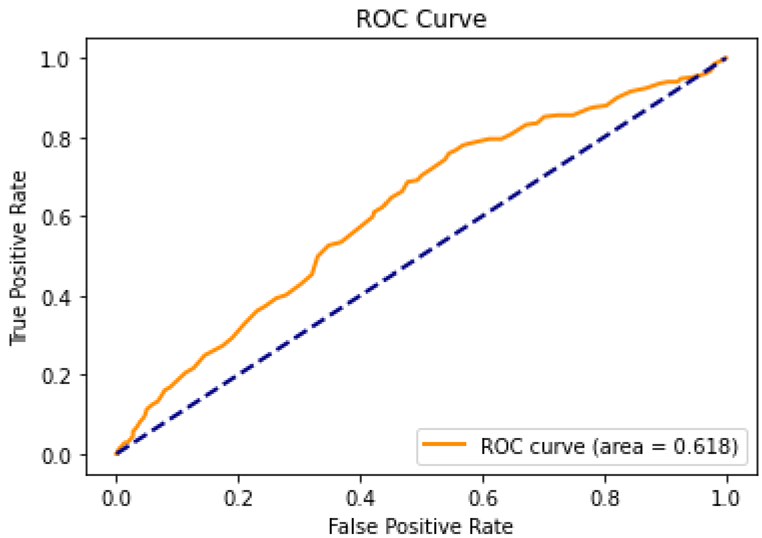
Below the **ROC curve** for model 1.

In [25]:
```
fprs, tprs, thresholds = roc_curve(y_true=df['y'],
                                   y_score=mod1.fittedvalues)
auc = roc_auc_score(y_true=df['y'],
                    y_score=mod1.fittedvalues)
print(auc)
```

```
0.6178832978974722
```

In [26]:
```
def plot_roc(fpr, tpr, auc, lw=2):
    plt.plot(fpr, tpr, color='darkorange', lw=lw,
             label='ROC curve (area = '+str(round(auc,3))+')')
    plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--')
    plt.xlabel('False Positive Rate')
    plt.ylabel('True Positive Rate')
    plt.title('ROC Curve')
    plt.legend(loc="lower right")
    plt.show()
```

In [27]:
```
plot_roc(fprs, tprs, auc)
```



5.2 <u>Model 2</u>: Logistic regression model that predicts approval of presidential foreign policy given age and sex.

Let's reread the data in again, as we would now like to model approval for the president's foreign policy with age, sex, and party.

In [28]:
```
missing_values = ["NaN", "nan", "Don't know/Refused (VOL.)"]
#
df = pd.read_csv('Feb17public.csv', na_values=missing_values)[['age','sex','q5
cf1','party']]
df.head()
```

Out[28]:

	age	sex	q5cf1	party
0	80.0	Female	NaN	Independent
1	70.0	Female	Disapprove	Democrat
2	69.0	Female	Disapprove	Independent
3	50.0	Male	NaN	Republican
4	70.0	Female	Disapprove	Democrat

In [29]:
```
df=df.dropna()
df.head()
```

Out[29]:

	age	sex	q5cf1	party
1	70.0	Female	Disapprove	Democrat
2	69.0	Female	Disapprove	Independent
4	70.0	Female	Disapprove	Democrat
6	89.0	Female	Disapprove	Independent
7	92.0	Female	Approve	Republican

In [30]:
```
df['y']=df['q5cf1'].map({'Disapprove':0,'Approve':1})
df.head()
```

Out[30]:

	age	sex	q5cf1	party	y
1	70.0	Female	Disapprove	Democrat	0
2	69.0	Female	Disapprove	Independent	0
4	70.0	Female	Disapprove	Democrat	0
6	89.0	Female	Disapprove	Independent	0
7	92.0	Female	Approve	Republican	1

In [31]:
```python
mod2 = smf.logit(formula='y ~ age+sex+party', data=df).fit()
mod2.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.419649
         Iterations 7
```

Out[31]:

Logit Regression Results

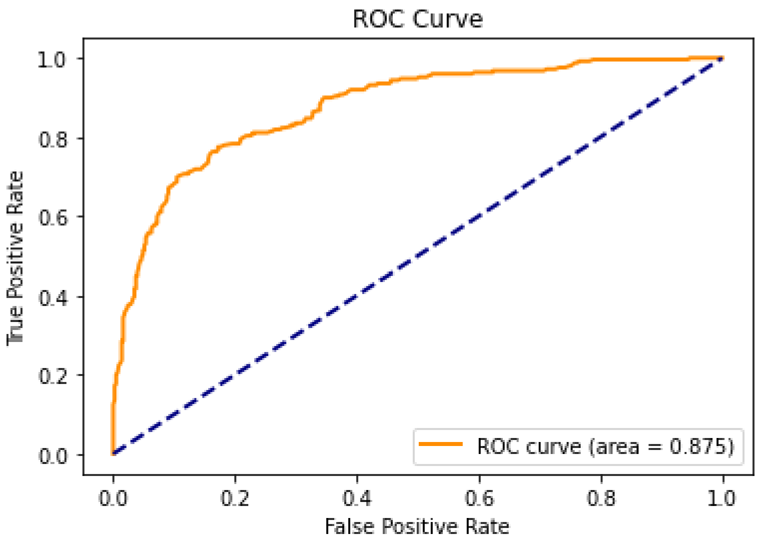
Dep. Variable:	y	**No. Observations:**	679
Model:	Logit	**Df Residuals:**	672
Method:	MLE	**Df Model:**	6
Date:	Thu, 08 Apr 2021	**Pseudo R-squ.:**	0.3614
Time:	13:28:07	**Log-Likelihood:**	-284.94
converged:	True	**LL-Null:**	-446.23
Covariance Type:	nonrobust	**LLR p-value:**	1.185e-66

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-4.5635	0.465	-9.807	0.000	-5.475	-3.651
sex[T.Male]	0.7288	0.217	3.363	0.001	0.304	1.154
party[T.Independent]	2.2604	0.312	7.236	0.000	1.648	2.873
party[T.No preference (VOL.)]	2.5881	0.680	3.808	0.000	1.256	3.920
party[T.Other party (VOL.)]	4.0865	1.212	3.372	0.001	1.711	6.462
party[T.Republican]	4.2985	0.341	12.592	0.000	3.629	4.968
age	0.0272	0.006	4.443	0.000	0.015	0.039

In [32]:
```python
fprs, tprs, thresholds = roc_curve(y_true=df['y'],
                                   y_score=mod2.fittedvalues)
auc = roc_auc_score(y_true=df['y'],
                    y_score=mod2.fittedvalues)
print(auc)
```

```
0.8750863920799477
```

In [33]:
```
plot_roc(fprs, tprs, auc)
```



STAT 207, Victoria Ellison and Douglas Simpson, University of Illinois at Urbana-Champaign

In [ ]:
```

```